Slices and Lumps: Division and Aggregation in Law and Life (Intro + Ch 1)

Lee Anne Fennell
CONTENTS

Introduction / 1

ONE / Surveying Lumpiness / 8

TWO / Assembly and Division / 27

THREE / Lumpy Goals, Segmented Resources / 45

FOUR / Increments and Incentives / 64

FIVE / Intrapersonal Dilemmas / 84

SIX / Saving and Spending / 102

SEVEN / Work, Play, Risk / 119

EIGHT / Buy, Own, Split / 136

NINE / At Home / 155

TEN / In the City / 172

ELEVEN / Law’s Cliffs / 190

TWELVE / Legal Bundles / 208

Conclusion / 227
Introduction

If you were a superhero, what would be your superpower? Flying? Invisibility? Time travel? I would pass up these familiar options in favor of the profoundly important but woefully underrated power of configuration—the ability to divide things up that arrive in lumps and to put things together that arrive in pieces. These feats might sound simple, but they are extraordinarily valuable and often maddeningly elusive.

To see why, think of all the things that might do you more good if they were sliced up differently. Perhaps you would prefer a job that involves a third less work and a third less pay, or a home that is half its size except when you are entertaining, or a car that materializes only when needed and is priced accordingly, or a dog that provides half the affection and requires half the walking. Next, think of the many things that arrive in fragments but that gain much or all of their value only when put together. The pieces necessary to build a complete rather than partial bridge. Votes to create a political result. The increments of studying necessary to pass a high-stakes exam. Patent licenses to produce a particular product. Or the bits of extra space between parallel-parked cars that you wish you could aggregate together to create a space large enough for your car. Getting part of the way there doesn’t always get you a proportionate share of the total benefit (think of a partial bridge or a partial parking space).

Superpowers throw human limitations into plain view, and a central goal of this book is to explore why reconfiguration is both important and difficult. Once we look carefully, we see that difficulties in slicing and lumping shape much of the way we have organized our lives, and a great deal of law and policy as well. From hot button issues like eminent domain and habitat conservation to developments in the so-called sharing economy (better termed “the slicing economy,” I argue) to personal
struggles over work, risk, money, time, diet, and exercise, how things are divided up or aggregated together matters tremendously. Understanding the nature of configuration problems enables us to deal more effectively with them. By exerting control over how things are divvied up or pieced together, individuals, firms, and governments can shape outcomes in every domain of life, law, and policy.

Configuration, in short, is power. It is a power that has become increasingly pressing to understand and harness. New technologies and growing urbanization have made it easier than ever to bring people together in both real and virtual space to share ideas, make new things, and join forces on projects of all kinds. At the same time, emerging forms of unbundling, from jobs to cars to homes to entertainment, have refined the slices in which we produce and consume. It is no exaggeration to say that the future of the city, the workplace, the marketplace, and the environment all turn on questions of configuration, as do the prospects for more effective legal doctrines, for better management of finances and health, and much more. Yet the art and science of configuration is not a recognized field of inquiry. This book aims to make it one.

By the end of the book, I hope to have convinced you of the power of configuration, and to have illuminated how indivisibility and fragmentation generate—and sometimes help solve—a wide range of legal and social problems. My inquiry uncovers some unappreciated and often surprising ways that the increments into which choices or resources are divided or aggregated can influence human behavior. This book highlights how governmental actors, markets, and households slice and lump (often in unacknowledged ways) and how they might do these things better. I offer strategies for recognizing and harnessing the power of slicing and lumping in law, policy, and everyday life. I hope to make configuration entrepreneurship salient—both as a focus of private and public innovation and as a crucial form of life-hacking.

The evocative economic concept of “lumpy goods” offers a starting point for my analysis. In a classic paper, Michael Taylor and Hugh Ward observe that some goods, like bridges and rail lines, “cannot be usefully provided in any amounts but only in more or less massive ‘lumps.’” Lumpiness sometimes refers to a desired end state, like the complete bridge. In other cases lumpiness represents an impediment to reaching a preferred end state—one wants only part of a job, say, or a share of a car, but (for whatever reason) the good is produced or provided in an all-or-nothing fashion. The inability to divide things up also limits the ability to make things
incrementally bigger. For example, production or computing capacity can often be added only in large chunks.

Some constraints are physical or technical in nature and may be surmounted, if at all, at great expense. For example, the Silver Spirit cruise ship, a 642-foot-long vessel in Silversea’s fleet, was recently cut in half to insert a new forty-nine-foot midsection that will add about 12 percent to its passenger capacity. This ship-splicing represents a rare engineering feat—one that will consume roughly 450,000 worker-hours—and its difficulty and cost attest to the inherent lumpiness involved. Evolving technology is making rapid inroads on other kinds of indivisibility, however, as we see with new platforms for dividing access to houses, cars, clothing, and more. Many other forms of lumpiness are intentionally constructed by government or private actors—minimum lot sizes or product bundles, for example—and thus represent potentially malleable features of social, legal, and transactional settings.

Despite the evident centrality of lumpiness and divisibility to law and policy, these concepts have received only scattered attention from legal scholars. This might seem surprising, especially given the prominence that the economic analysis of law enjoys. But economics itself also tends to neglect these matters. This is partly for reasons of mathematical simplicity—models are more tractable if a linear relationship between inputs and outputs is assumed. And in the large-number settings that much economic analysis focuses on, indivisibility is not especially consequential: for a factory making hundreds of widgets per day, it hardly matters that producing each widget is an all-or-nothing proposition. Moreover, economists have long recognized that although individual decisions may be lumpy—a stable owner cannot reduce his team by a fraction of a horse when oat prices rise slightly—markets as a whole exhibit what Andreu Mas-Colell calls “the regularizing effects of aggregation.” At a large enough scale, lumps come out in the wash.

Yet for individuals—workers, consumers, household members, risk bearers, taxpayers, and citizens—lumps matter profoundly. As Hagan Bobzin observes, making one more car “is of little significance for an automobile company, whereas a household faces considerable consequences depending on whether it has got a car or not.” People cannot successfully navigate the interactions that are most important to their lives without at least an intuitive understanding of the significance of slicing and lumping. For related reasons, law and policy cannot afford to ignore matters of configuration. Not only is legal analysis frequently concerned with the structure of
individual decisions, but social policy regularly addresses unique, indivisible goods and large-scale goals that are not amenable to the marketplace’s alchemy of averaging.

Take conservation, a context in which recognizing lumps of value can upend established ways of pursuing goals. Mary Ellen Hannibal recently observed: “For more than one hundred years, conservation has functioned by drawing a boundary around a special area and limiting human impacts there . . . . But science today tells us this approach is failing. Nature doesn’t work without connection.” In other words, the world is lumpy, and some of the most significant lumps of value may not correspond to the ways in which resources like land have traditionally been sliced up. This reality is now being recognized through efforts to create migratory pathways and wildlife corridors. Here, as in other contexts, it is impossible to devise meaningful solutions without appreciating the lumpiness lurking in natural and social phenomena.

Lumpiness can also produce or explain behavior that seems to defy basic economic principles. For example, the law of diminishing marginal returns suggests that the next unit of a good will add less value than the previous unit. Lumpiness inverts that relationship: at times, one needs more of something to get any return at all. The lumpy or fragmented features of a given situation may also elicit behavior that is mistakenly attributed to behavioral biases. For instance, a person who plays the lottery or elects a lump sum over a larger payment stream may not be irrational or myopic, but rather simply expressing a strong preference for a lumpy consumption experience that is difficult or impossible to attain in any other way. Paying attention to configuration forces us to rethink our assumptions.

This is an especially exciting and crucial time to be studying questions of slicing and lumping. As increasing urbanization and environmental threats raise the stakes for land configuration choices, a technology-fueled entrepreneurial explosion is underway that is dividing goods, services, and jobs in novel ways, from Airbnb to Zipcar. This book highlights the connections between these and other social and economic developments, and examines the opportunities and concerns they present. It also sheds new light on chronic intrapersonal struggles, from overeating to the management of time and money, as well as persistent legal and policy puzzles, from the best way to deliver benefits to the best way to address risky behavior.

A few words about the book’s methods and goals will help to frame what follows. My approach here is primarily analytic. I seek to understand and explain configuration problems, to get inside them and see how they work, rather than advocate for particular solutions to them. Yet in so
doing, I mean to shed light on the ways that configuration matters to human well-being, and on the potential for better configurations to improve our lives. This book emphasizes the significance of the lumps and slices we encounter, and the need for our analyses and habits of thought to account for them. But this does not mean we must accept configurations as we find them. Even when indivisibilities arise from ecological or other natural phenomena, human reactions to them are malleable, making configuration an active enterprise, not a static fact. The words in my title are verbs as well as nouns.

For concreteness, my exposition is intensely example driven. There are large and deep literatures attached to many of the specific contexts I touch upon, which I cannot do justice to here. My aim is not to offer a comprehensive analysis of each of these situated examples, but rather to highlight the common structure they share—a forest that has been largely ignored in favor of individual trees. The book thus engages in a type of meta-lumping by highlighting connections and commonalities among diverse configuration challenges that have previously been treated in isolation. At the same time, this book distinguishes problems involving lumpy or indivisible goals or goods from the other types of collective action problems that tend to dominate the popular and academic imagination—a form of meta-slicing.

The first four chapters of the book lay the conceptual groundwork, starting with an overview in chapter 1 of the types of indivisibilities that appear in markets, communities, personal life, and law. Chapter 2 shows how lumpiness arises in high-profile contexts like eminent domain, which involves the forcible assembly of land, as well as in settings where resources that are currently co-owned must be split up among claimants. I show that these two types of problems—assembly and division—are not distinct, as is usually assumed, nor is one inherently harder to solve than the other. Instead, they share a common structure: each type of reconfiguration requires both assembly (of consent by the affected stakeholders, or an overriding of their lack of consent) and division (of the surplus that is thereby created). In both cases, what is really being pieced together—whether voluntarily or through coercion—is cooperation in pursuit of a lumpy goal, the resource’s reconfiguration.

Chapter 3 extends this theme of assembling cooperation to collective action problems more broadly, whether saving a fishery from collapse or collecting funds to cure a disease. I show how lumpy social goals—ones that are all-or-nothing—present different, and generally more favorable, prospects for success than the standard tragedy-of-the-commons scenario.
Also significant are the ways in which the resources to be harvested or the tasks to be contributed are divided up. Chapter 4 then considers how choice menus—whether sizes of sodas or technologies for fighting pollution—affect behavior by defining the increments in which people can take actions. When alternatives are chunky rather than continuous, people often must produce or consume either less or more than they would prefer—with overlooked and sometimes surprisingly positive implications for behaviors that have spillovers on others.

Chapter 5 turns to the ways in which aggregation and division impact intrapersonal dilemmas. Many of the same considerations that we observe in collective action problems among different people also apply when the players are different versions of oneself. Likewise, the chunkiness of the choices one encounters can edge decisions closer to one’s overall long-term interests or push them further away. Finding ways to strategically engineer and personalize choice menus offers new avenues for addressing self-control problems. Chapter 6 extends these ideas into the realms of personal financial management and public finance. Recognizing the significance of aggregation and division in saving and spending can improve how households manage their budgets and how governments formulate taxes, incentives, and benefits.

The next four chapters show how aggregation and division crop up in several important domains: the workplace, the marketplace, the home, and the city. Transformations are underway in all of these settings. Chapter 7 explores how new business models that slice time, effort, attention, and risk in unprecedented ways are changing how people work and play. The gig economy represents one manifestation of this shift, and the ambivalence surrounding it can be understood in terms of lumpiness: delumping the working experience has also meant decoupling work from many of its standard accompaniments, including health insurance. Chapter 8 examines the developing slicing economy in the marketplace for products and services. Here I explore the prospects and limits of swapping full-strength ownership for on-demand access. I also show how indivisibilities crop up in product bundling, sizing, pricing, and standardization, with implications for consumer choice.

Chapter 9 turns to housing, where innovative new forms of slicing abound, from platforms like Airbnb to social housing designs that deliver partial homes. At the same time, legal and policy choices often contribute to a discontinuous, chunky menu of housing alternatives that omits or limits options that people might prefer—such as very small units suitable for one-person households. Analyzing this constructed form of lumpiness
in housing raises questions about the scope of the home, ones that require examining complementarities between individual dwellings and the surrounding community. Chapter 10 widens the viewfinder to take in the city, where the questions of land assembly that appear early in the book are reconsidered in connection with agglomeration benefits (urban vitality) and costs (congestion). Perhaps the most pressing economic question of our day is how to make the most of our cities, which are themselves a paradigmatic instance of the power—and challenges—of aggregation.

The final pair of chapters extends the analysis of aggregation and lumpiness into legal decisions and doctrines. Chapter 11 begins with the observation that law often constructs cliffs or generates all-or-nothing outcomes. For example, judicial decisions are very often binary in nature (one party wins entirely and the other loses entirely). Messy facts drawn from a continuum of possibilities are rendered into all-or-nothing outcomes. Much turns, then, on the “thresholding” processes that the law uses to generate these on-off results. Questions of aggregation play a decisive role: a momentary lapse of judgment, for example, might fall on one side of a legal line if viewed in isolation and on the other if considered as part of a larger pattern of careful or careless behavior. Chapter 12 shows that many legal and policy debates boil down to disagreements about bundling—whether of precautions, property interests, behavior, regulations, or legislation. Because the power to bundle or unbundle can dramatically change results, battles over bundles are some of the most interesting and consequential disputes in law and policy.

The book concludes with takeaways for policy makers, lawyers, academics, and anyone else who is interested in understanding and leveraging the lessons of lumpiness. Issues of lumpiness and divisibility touch nearly every corner of human experience, and they offer countless opportunities for innovation and entrepreneurship. Although the contexts I cover are necessarily illustrative rather than exhaustive, I hope that this book will spur others to identify additional arenas where the ideas explored here can be applied and extended. There are, of course, many other ways that the terrain I cover could have been broken up and heaped together. But I hope that the current configuration will let through enough light to intrigue you, and to inspire your own efforts at lump building.
ONE

Surveying Lumpiness

Picture a bridge spanning a chasm. Removing one chunk of the span renders it worthless—indeed, it is no longer even a bridge. Because bridges are useless unless they are complete, they offer intuitive examples of lumpy, indivisible, or “step” goods. Lumpiness is found not only in large-scale infrastructure like bridges, highways, and railroad lines, but also in ordinary products and services. Some goods, like car tires or developable land, are more valuable if consumed in particular quantities or combinations. Others, like cars, jobs, houses, and pets, are often available only in difficult-to-divide chunks. Conditions like species survival or election wins depend on maintaining or reaching critical thresholds, not merely coming close. Legal rules and litigation outcomes may also exhibit lumpiness, operating in an all-or-nothing fashion, or producing results only when some threshold of compliance or deterrence is reached. And the lumpy fixed costs that attach to many endeavors—from introducing a new product to passing a new law to learning a new skill—make choices fewer and chunkier for firms, consumers, citizens, and workers than they otherwise would be.¹

These and many other examples will be explored in the chapters that follow. Here, I take up two foundational questions: What counts as “lumpy”? And why do we care? The answers to these questions will preview the range of aggregation and division problems taken up in this book. Many of these problems involve desired, attempted, thwarted, or contested reconfigurations—attempts to slice up things that are difficult to divide or to aggregate things that start out in pieces. Others concern the appropriate legal or practical treatment of naturally occurring or constructed lumps, whether in regulatory policy, legal analysis, informal order, bargaining settings, or the realm of self-control.
What's Lumpy?

The idea of lumpiness seems intuitive, but the term is used in more than one way and encompasses a variety of phenomena. Some distinctions and definitions will help to set the stage.

Supply, Demand, and Lumpiness

We might refer to a good as a lumpy or indivisible either because this is how the good delivers its value (in a lump, like a bridge) or because the good arrives in a lump and is accompanied by constraints (natural or constructed) that make it difficult or costly to divide (think of the full-time position that does not allow for part-time work). These are, in a sense, opposite meanings. In the first, lumpiness describes a desired end state (the completed bridge). In the second, lumpiness describes a suboptimal starting point (the full-time job). In both cases, there is a mismatch between the starting point and the desired end state, but what is necessary to span that gap differs. To build the full bridge, many smaller pieces must be assembled. The lumpy job comes preassembled, and that is exactly the problem—a slice of the job would be preferable for the employee.

One way to express this distinction is between goods that are lumpy in demand (people want full bridges) and goods that are lumpy in supply (cars and pets come in whole number units). Some goods might be described either way. For instance, we could say that an employer supplies jobs in full-time increments or demands labor in full-time increments. Regardless, lumpiness becomes interesting where what is desired (by someone) takes a different form than what is provided (by someone else). A good that is lumpy in demand, like a bridge, often must be assembled from inputs—bridge segments, labor, financial contributions, and so on—that are fragmented in supply. A good that is lumpy in supply, like a car, may need to be split into smaller use-slices to effectively meet consumer demand.

Often lumpiness is of no consequence because it can be addressed through ordinary markets or informal transactions. For example, if the smallest unit of candy that can be economically produced and sold separately is a 1.5 ounce candy bar, and if most people have no desire to purchase candy in smaller increments than this, whatever theoretical lumpiness may exist presents no difficulties. Lumpiness becomes problematic when the supplied units are much larger or smaller than desired (think of a mammoth candy bar or a single chocolate chip) and there are significant
impediments to dividing up the larger unit or aggregating the smaller ones. The obstacles may stem from physical constraints or the costs of engaging in market transactions. They may even be social or psychological in nature. Philip Henry Wicksteed, writing in 1910, observes that the commercial standard of supplying ink in one-penny measures effectively precludes people from acquiring smaller quantities, given the “awkwardness and humiliation” involved in negotiating with a stationer for a smaller amount.

Lumpiness can also cause difficulties when everyone agrees that the initial (lumpy) configuration is the most valuable one, but there is more than one plausible claimant. A vivid example is the dispute over the baby that featured in King Solomon’s famous decision. Babies, it turns out, are extremely lumpy. Luckily, there are alternatives to physical division, and the Solomonic outcome illustrated one of them—an award to the claimant who clearly valued the child more. As the literature on this topic has noted, indivisibilities may be addressed through a variety of techniques, including slicing the good temporally (e.g., through rotation systems); converting the good into something divisible like money, as by auctioning it off; giving claimants chances at the good that are proportionate to the strength of their claims; or giving the good to one claimant while compensating the others.

Temporal slicing of goods is an especially intriguing solution because it can bridge the gap between the physical configuration that maximizes value and the amount of the good that a particular individual wants, needs, or is entitled to receive. It works well for goods that are far more valuable when physically intact, where people do not want, and are unwilling to pay for, the whole thing. No formal slicing is necessary if people can agree to share the resource. In some cases we manage to do exactly that. People form clubs or enter communities to consume certain kinds of indivisible goods—swimming pools, tennis courts, clubhouses, and so on. Other varieties of time slicing are longstanding and familiar: library books, hotel rooms, rental cars, and so on. Entrepreneurs are now finding a multitude of ways to create small-scale market transactions that further fine-tune slicing, as evidenced by Airbnb, Uber, and many other business models. An extreme example is Recharge, an app that allows people to buy “microstays” at hotels and apartments, priced by the minute.

Consider another innovation in temporal slicing, pet sharing. Companion animals, like babies or bridges, are lumpy and can’t be physically divided. But the unit in which pets arrive is not necessarily the optimal unit in which their companionship is consumed. Suppose that for one individual, Angus, dog ownership is great fun for a few days a week, but the
Surveying Lumpiness / 11

burden continues to grow as the week wears on, and the benefits diminish apac. If the unfun days of Angus’s dog-owning week could be transferred to other people who similarly experience declining returns from dog ownership (Beth and Cam, say), the dog could deliver a larger total quantum of enjoyment to its (now plural) owners.

There may be problems, of course. Time-share dog owners may shirk on bathing the dog or taking him to the vet. The dog may never get properly trained, or the constant parade of owners may produce anxiety or confusion for the dog. Some of these issues might be overcome by, for example, having a platform manager who coordinates tasks, establishes minimum time blocks, and sets care standards, but these solutions add to the costs of time slicing. BorrowMyDoggy.com, which currently operates in the UK and Ireland, enables a pet owner who retains primary responsibility for her pup to offer short-term “borrowing” in exchange for dog walking, care, or socialization, while the platform provider collects a fee that covers veterinarian access and insurance. This model offers an approximation of informal interactions over pets among friends and family, adapted to urban settings where people often lack preexisting social networks. Here, as in many other contexts, from ride sharing to home sharing, we see new models for managing lumpiness emerge as earlier (and mostly unremarked) ways of informally aligning supply and demand break down.

**Some Terminology**

The notion of lumpiness connects tightly to the concepts of *indivisibility* and *complementarity*. To say that a good is indivisible or that it exhibits indivisibilities does not usually mean that the good literally cannot be divided, but rather that it is considerably less valuable when divided, or that it is expensive (perhaps prohibitively so) to divide successfully. The idea of complementarity refers to the fact that certain goods and services produce more value when consumed in particular combinations. Right and left shoes are a standard example. Because most people have two feet of similar size and follow the social custom of shodding them identically, a pair of shoes typically delivers far more than twice as much value as a single shoe. Likewise, the segments that make up a full bridge span are strongly complementary; subtract just one, and the bridge becomes useless. A partially fenced yard does no better than an unfenced yard at containing animals, a car with three tires drives no better than a car with no tires, and small and scattered patches of land are useless for large-scale development.

In these familiar examples, indivisibilities are a function of comple-
mentarities. A set of tires or a pair of shoes exhibits indivisibilities not because tires or shoes are physically hard to separate from each other, but rather because splitting them up would be self-defeating—they are much more valuable when consumed together. Not all indivisibilities track complementarities in this way. Other things that we might characterize as indivisible (cars, jobs, pets, houses, and so on) might be more valuable in pieces (whether time slices or physical slices) but dividing them up is for some reason technologically or administratively difficult. I will use the term indivisibility in this book as a synonym for lumpiness. The notion of complementarity represents a general purpose explanation for why goods or services might be more valuable when aggregated in certain ways.

Two other terms associated with lumpiness are discontinuities and nonlinearities. Returns from activities like studying or voting are often discontinuous: making it over some threshold makes the difference between passing and failing, or between winning and losing an election. Nonlinearities occur when outcomes do not increase smoothly and proportionately in response to inputs. There may be increasing returns (economies of scale), diminishing returns (diseconomies of scale), sharp steps or notches at particular thresholds, or some mix of these effects. The economic tool of the production function, which maps inputs to outputs, provides traction on these ideas.

**Lumpy Production Functions**

Lumpiness can be understood as a certain kind of relationship between inputs (units of effort, money, or resources) and outputs (conditions, events, products, or services). Consider, for example, the connection between dollars contributed to a charity and the benefits that the charity generates in the world. If this relationship is plotted on a graph with well-being improvements on the vertical axis and dollars on the horizontal axis, what shape will the curve take?

There are many possibilities. Perhaps the relationship is linear, at least within a particular range, so that each additional dollar generates the same uptick in benefits. Think of assistance that buys increments of soup, medical care, or clean drinking water, which in turn produce a corresponding improvement in well-being among the recipient population. In other cases, a plateau may be reached after which additional dollars do less good than the dollars that went before—after every household has mosquito nets, say, the next best uses of the money may be less effective at producing marginal improvements. Conversely, there may be a snowball effect, so that
as more contributions are added, each does more and more good, at least up to a point—think of class sessions added to an educational program, or inoculations against communicable diseases within a community. Or the curve may be S-shaped, with a range of increasing effectiveness followed by a range of diminishing returns.\(^{14}\)

Production functions for lumpy goods deliver outputs not in smooth, regular increments as individual units of input are added, but rather in large jumps after a series of inputs.\(^{15}\) At the extreme is a pure step good that delivers all of its utility in one large chunk or “step.” Think again of a bridge. Suppose you need to span a chasm that is a thousand yards long, and the bridge material arrives in one-hundred-yard segments.

As shown in figure 1.1, value to users remains flat as the first nine segments are added, one by one. But when the tenth unit is added to create a completed bridge, suddenly value steps up all at once. There is a sharp discontinuity, illustrated by the dashed line in figure 1.1. The step not only marks out a threshold under which no benefits are provided, but also represents a plateau from which no further incremental improvements are possible. Adding more lengths to the bridge once the span is complete does no good.

In fact, such pure step goods are rare. Even a bridge can be supplied at many different quality levels, as Russell Hardin has noted.\(^{16}\) An election is also a common example of a step good—here, the inputs are the votes that either do or do not reach the critical point that enables one’s preferred
candidate to win. Votes short of the amount necessary to win are useless in generating the desired outcome, while extra votes beyond that level are superfluous. Of course, if one defines political objectives slightly more broadly than choosing a winner in a particular contest, the step function looks less sharp. Often we think that landslides produce at least somewhat better results for the winner than do narrow victories, while near-misses provide greater political impetus for another try than would a crushing defeat. Nonetheless, these examples provide an intuitive sense of what a lumpy or step good looks like.

Equally rare are perfectly linear goods—those with a smooth, continuous production function in which each infinitesimally fine unit of input is matched by a corresponding adjustment in output or utility. Few products can be produced, purchased, or enjoyed in literally any quantity. Often some minimum threshold must be crossed to obtain (or enjoy) the thing at all, and many goods must be transacted over in integer units (bananas, for instance). Even readily divisible goods—Wicksteed uses the example of pudding servings for children—may be relatively valueless below a certain quantity threshold.

Between the extremes of a perfectly linear good and a single-step good, we find different degrees of nonlinearity or indivisibility. Consider figure 1.2, which depicts an S-curve. This curve corresponds to a relatively lumpy good that does not take a pure step form.

Although this good does not deliver all its value in a single shot, its pro-

![Figure 1.2. The S-Curve. Source: Fennell, “Lumpy Property,” 1960, fig. 2.](image-url)
duction function contains ranges over which the marginal effect of added pieces is sharply increasing or decreasing. The S-curve describes many collective goods that require a critical mass of participation to succeed, but that at some point plateau.²⁰ It might also fit with certain kinds of land assembly projects, where value increases sharply once a certain number of parcels are aggregated, but where having all the parcels is not essential.²¹

Lumpiness, as used in this book, refers to severe discontinuities or nonlinearities in the production function, whether or not those functions take a pure step form or intersperse segments of sharply increasing or decreasing returns with ranges exhibiting linearity.²² These differences in shape are important, however, because they can influence the prospects for cooperation and the risks of strategic behavior, as we will see in chapters 2 and 3.²³

**What’s in the Lump?**

So far, I have spoken of “segments” or “pieces” that produce value when aggregated together. Lumpiness or indivisibility often refers to quantities of relatively fungible inputs—segments of a bridge, lengths of railroad track, tires for a car, units of work, and so on. Yet it may also refer to systems made up of heterogeneous elements, such as a machine that cannot operate without each and every one of its parts.²⁴ I will use the notion of lumpiness broadly and functionally here to refer to both heterogeneous and homogeneous aggregations, given that both forms of lumpiness can generate similarly structured problems.

In the context of land assembly, for example, the unique spatial location of each parcel makes the component parts of the desired assembly unique and nonfungible. But this sort of nonfungibility is neither necessary nor sufficient to produce an assembly problem. Even if a group is building a bridge out of identical, interchangeable segments, there may still be an assembly problem if there are no outside sources of bridge material and each individual in the group holds a segment essential to the whole. Conversely, a car may require many different mechanical parts to run (none of which could substitute for each other), but there will be no difficulty assembling the necessary pieces as long as each part is readily available on the open market. The car is still lumpy in that its parts are interdependent and all of them are needed, but this lumpiness may pass unnoticed as long as the underlying markets for its inputs remain competitive. What matters most to the shape of an assembly problem, then, is not whether the necessary components are interchangeable with each other, but rather whether close substitutes exist for each of the components required for a given assembly.
As this example suggests, the lumpiness of a good or service is distinct from the market conditions that influence whether, or how easily, the full lump will be obtained. Familiar examples of lumpiness, like large-scale developments that require assembling many pieces of contiguous land, often confront the holdout power held by the various owners of the component parts. But the lumpiness of the project would remain (though it would likely go unremarked) even if the property were all initially held by the developer. Thus, lumpiness tends to announce itself as such when some impediment stands in the way of achieving it or breaking it down—whether monopoly power, technological limits, or other factors.

The components making up a given lump may also be segments of time. Some goods, such as private residences, can become disproportionately valuable when consumed over lengthy, unbroken periods. Often, the most valuable temporal chunks are defined by reference to external events, such as the length of a life, a job, or an educational program, or the time that it takes for a particular risky investment to yield returns—all of which can be uncertain. Property rights that let owners hold onto things long enough to realize distant or uncertain payoffs respond to this temporal lumpiness.

Finally, sometimes lumps represent not the way in which goods generate value, but rather technological or natural constraints on how goods are produced or supplied. As we have seen, it is possible to have goods that are lumpy in supply that become more valuable when divided (temporally or physically) among different people. This kind of lumpiness indicates a discontinuity or nonlinearity in the production process, perhaps due to high fixed costs or other economies of scale. Once the good is supplied, the challenge is to come up with a plan for dividing the consumption experience. Indeed, without a plan for dividing the consumption experience, the good may not be profitable to supply in the first place.

Subdividing Lumpiness

To get a better sense of the scope and variety of lumpiness-related issues that crop up in the real world, and to more clearly see what is at stake, it is helpful to consider some other ways of subdividing the category.

Goods and Bads

So far I have spoken of lumpy goods. But sometimes an undesired end state takes a lumpy shape. Russell Hardin gives the example of a power blackout to illustrate a step bad: the blackout will occur all at once if aggregate...
electricity usage exceeds a critical threshold.\textsuperscript{25} Or consider a population crash that will cause the demise of a fishery.\textsuperscript{26} Below some threshold of harvesting, nothing much changes, but once the threshold is crossed, disaster ensues—a lumpy bad. When the goal is to avoid a catastrophic end state (rather than to achieve a desired end state), the challenge becomes one of limiting the piecemeal acts of harvesting or destruction that can combine to push beyond the threshold of sustainability. This challenge can be recast as one of assembling forbearance from those who are otherwise entitled to draw from the common supply. Assembling enough forbearance to keep the lights on or pull the fishery back from the brink can be recast as achieving the lumpy good of “avoiding a blackout” or “avoiding a population crash.”

In other cases, what is a lumpy good for some people is a lumpy bad for others. Göran Bostedt analyzes the case of the Swedish wolf, whose preservation constitutes a public good for many Swedish nature lovers, but a public bad for reindeer herders whose herds suffer depredation from the wolf.\textsuperscript{27} Although it is possible to have more or fewer wolves, if the population threshold that is robust enough to satisfy the wildlife lovers also creates a serious threat to the herders, this is a lumpy state that will be sought by some and opposed by others.\textsuperscript{28}

Differences of opinion may also emerge as to whether a given resource is more valuable when split up or when maintained as a unit. For example, what appears to be a problem of lumpiness in supply (a pet or job that cannot be divided in half) may in fact be the most efficient arrangement. Maybe nobody wants to sign up for half of your dog or the last three hours of your job each week, at least not at a price you would find agreeable. Splitting the resource might, in fact, destroy rather than create value. In these cases, the key question is whether there is any gain to be had by reconfiguring—and the answer may turn on private information about valuations, as the next chapter discusses.

\textbf{Natural versus Constructed}

There is little mystery why bridge crossers demand a full rather than partial bridge—they are susceptible to gravity, and this fact about the physical world is reflected in the lumpiness of bridge structures. Likewise, a lumpy bad like the extinction of a species turns on an ecological reality, the threshold at which overhunting or habitat loss will render the population unsustainable. In these examples, lumpiness stems from naturally occurring discontinuities. In other cases, lumpiness is a function of techno-
logical constraints, such as a Coke machine that can take only nickels, or a manufacturing process that requires a minimum production run to cover high fixed costs. In yet other cases, the lumpiness is constructed by law. For example, a square footage minimum for apartments makes housing lumpy for reasons that stem not from physical or technological limitations on construction, but rather from (often contested) societal judgments.

Both private parties and policy makers may intentionally construct lumps that are hard to break apart in order to force people to make choices that are bundled, take-it-or-leave-it propositions. For example, the inability to negotiate over boilerplate terms in a lease or contract has the effect of making the leasing or contracting decision lumpy. Even seemingly mundane decisions about the size or quantities of products can influence choices quite profoundly. Think of sugary sodas or cigarettes—goods often viewed as bads when consumed to excess. The inability to choose one’s preferred size or quantity of these items might result in reduced consumption for both psychological and economic reasons—or, alternatively, could make matters even worse (if, say, one buys multiples of a smaller size that amount to a larger total). Counterintuitively, even very large sizes might at times be part of a strategy to reduce consumption, if it puts people to an all-or-nothing choice in which “all” is unpalatably large.

An especially interesting form of constructed lumpiness involves property rights. To what degree do these rights correspond to cohesive “things” (from which the owner can categorically exclude others) rather than bundles of entitlements that are either endlessly flexible or at least socially and culturally contingent? Henry Smith puts it this way: “Property organizes this world into lumpy packages of legal relations—legal things—by setting boundaries around useful attributes that tend to be strong complements.” Property ownership characteristically structures access to these presumptively complementary resources through a block of delegated control that excludes the uninvited and extends unbroken through time, bundling access today with access tomorrow and tomorrow and tomorrow. This setup allows people to reap what they have sown (both figuratively and literally) and to hold onto the land or other asset as long as necessary to see returns on their investments.

But property is not just lumpy; it is also sticky. Attributes that were at one time complementary may tend to remain together as chunks of ownership (an entire car, say) long past the time when they continue to generate more value aggregated than disaggregated. New business models that offer thinly sliced rights in resources—from rides to tools to toys to lawns to clothing—highlight the inherent lumpiness in traditional property owner-
ship, as well as the possibility of alternatives. In other words, if the strong complementarity that originally made us draw property lines here and not there is contingent on social, cultural, and technological factors, rebundling becomes necessary as conditions change. And we are currently seeing a groundswell of changes along just these lines as new ways of slicing up access to goods and services become central to everyday life.

Property, then, provides an especially compelling setting in which to consider foundational questions about natural and constructed lumps of value. Do particular sets of rights (or particular physical or conceptual objects) possess some inherent unity that resists splintering, or are lumps instead largely of our own making? Gregory Alexander has recently explored similar questions in considering parallels between the “thing-ness” of works of art and of property—both of which can prove much less stable than is often assumed. Those same concepts can assist in examining other legally or socially constructed lumps to see whether they correspond to valuable complementarities that should be maintained or whether they are merely artifacts of past complementarities that exist no longer.

More broadly, the issue of composition—when (and whether) components may be said to form a coherent thing—is a subtle and philosophically interesting one. Peter van Inwagen presents a thought experiment in which people believe they are seeing black tigers or “bligers” in the distance, when in fact they are seeing sets of six separate animals—four monkeys, a sloth, and an owl—moving in concert so that they appear to compose single creatures. As the bliger tale suggests, the fact that components are in contact with one another does not necessarily make them part of the same organism. Conversely, what might look like many separate entities may instead be a single thing. Consider Pando, an aspen forest in Utah made up of an estimated forty-seven thousand genetically identical trees joined by a shared root structure, which is reputed to be the planet’s largest living organism.

Even when entities are intentionally constructed, questions remain about what is inside and what is outside. Ronald Coase famously explored the boundaries of a firm by considering the relative costs of conducting transactions inside and outside the envelope of the business entity—the make-or-buy decision. In urban contexts, the question of what counts as part of the same city can have more than one answer depending on whether one is referring to jurisdictional boundaries or functional interactions. Yet even the former is open to redefinition, as can be seen in a plan to split Sydney, Australia, into three separate cities.

Law too must often make judgments about what counts as part of the
same entity or event. What counts as a single crime, for example, and when does it begin and end? Should a person’s past pattern of conduct be relevant in a tort action, just the moment that caused the accident, or something in between? Is a person’s whole life or some smaller slice the relevant unit when assessing inequality, pursuing societal well-being, or setting tax policy? Similar aggregation questions run through all of law.

Rival versus Nonrival

Another dimension for classifying indivisibilities involves the distinction between rival and nonrival goods. Certain goods like ideas, songs, landscapes, and lighthouses do not get used up as people consume them—this makes them “nonrival” in consumption. Nonrival goods are inherently lumpy in supply: supplying such goods for many people costs no more than supplying them for one person. Nonrival goods are frequently lumpy in a more familiar sense as well: they cannot be enjoyed at all until a certain threshold is reached and gain nothing from inputs beyond that level. As Fred Thompson explains, “half a lighthouse is, perhaps, worse than useless, more than one is redundant.” These lumpy or “discrete” nonrival goods are effectively one-offs; only a single unit of the underlying good is ever produced. They are all-or-nothing propositions, where the question is not how much to produce, but whether to produce the thing at all.

Two opposing observations highlight the complex role of indivisibility in this analysis. First, lumpy nonrival goods can be easier to supply through voluntary cooperation since anyone (or any set of anyones) who cares enough about consuming the discrete good should be willing to underwrite its production, even though others will benefit. The fact that others will benefit may produce strategic behavior—everyone would prefer to have others fund the good while enjoying it for free—but people may still find contributing to be in their rational self-interest.

Second, and cutting in the other direction, nonrivalry disables the most intuitive basis for divvying up access and payment among users: consumption. Because my eating a pint of berries precludes you eating the same pint of berries, it seems only natural to charge me for the berries that I wish to eat and to assign me exclusive rights in those berries. I am getting what I paid for. Yet it is probable that the berries would be produced at exactly the same scale even if I did not buy my marginal pint. It is unlikely my purchase caused the berry patch and workers’ hours to be incrementally expanded exactly one pint’s worth. Instead, investments in berry produc-
tion are large scale and lumpy, but the units of berries are priced to cover the production costs. Seen in this light, the nonrival good does not seem much different—there is still a lump of production to fund—but because the most intuitive basis for assessing payment obligations is absent, another funding approach is required. Chapter 3 will consider this question further.

Types of Lumps

We can round out our survey of lumpiness with a nonexhaustive list of categories in which indivisibilities in supply or demand can be found, including goods, services, events, conditions, goals, and laws.

Goods

As we have seen, consumer goods may be offered in indivisible units, such as an entire car or an all-the-time pet, when some consumers would prefer smaller increments of ownership, such as a car for weekday mornings only or a pet that is one’s own only on alternate weekends. Similarly, purchasing a minivan or a three-bedroom home means owning the full structure all of the time, even if a vehicle half as large would suffice for the majority of car trips and the third bedroom is only used a dozen days each year. Firms and other large organizations like universities face related constraints: expansions in capacity may be available only in relatively large increments (a new plant, a large chunk of network capacity, or a new building), producing a forced choice between inadequate capacity and capacity that will appear excessive, at least in the short run. 47

Another aspect of lumpiness in supply, recently explored by Joel Waldfogel, relates to the fixed costs of production, which can limit the variety of goods produced. 48 Here, the problem is not that individual customers are forced to purchase more of a good than they desire, but rather that consumers must collectively purchase a threshold amount of a given good in order for its manufacture to be cost justified. Changes in the technologies of production and distribution have enabled a larger set of consumer preferences to be served in many markets, 49 but those with nonmainstream tastes may still find themselves out in the cold, especially for goods and services that must be consumed locally and thus cannot draw on a larger market. For example, commercial airline routes serving particular cities depend on a critical mass of passengers for their viability—a fact that has led to federal subsidies for service to smaller communities. 50
We have seen that goods often must be consumed in particular quantities or combinations in order to deliver value—whether matched sets of shoes or tires, or complementary goods like printers and ink cartridges—a fact that presents few difficulties if the relevant markets are competitive. But when monopoly power exists over some or all of the components, aggregation can become difficult. Land assembly is a special case of this general problem. Similar issues exist for products or creative works that depend on inputs to which others hold intellectual property rights.

**Services**

Like the manufacturer who has a minimum efficient size for a production run or product, entities or persons providing services may not be willing or able to supply those services in minutely divided segments. For example, because a plumber cannot make half a service call, the amount paid for the call must cover the cost of time and vehicle use necessary to actually complete the call. To be sure, the plumber can do greater or lesser amounts of work while out on the call, can use more or less expensive materials, and greater or lesser amounts of skill. But the client must at least cover the cost of getting out to the site and spending some minimum amount of time there or the service input will not be made at all.

The flip side of lumpy service inputs is lumpy service requirements. Here, think of the many young lawyers who complain that they would prefer to work somewhat shorter hours for lower pay, but find this alternative unavailable to them at major law firms. Here, the operative lumpiness may have little to do with the indivisibility of their own inputs—many could, in fact, easily work 10 or 20 percent fewer hours. Rather, the problem is that their employer requires a certain minimum amount of service in order to offer them jobs at all. If they fail to put in the requisite hours, the result is not a proportionately downscaled salary, but rather withdrawal of the employment opportunity altogether. The indivisibility in service requirements may be driven by the economics of hiring, training, and offering benefit packages to larger versus smaller numbers of workers. In some cases, however, such indivisibility may be artificially constructed by firms in an effort to screen out workers who are less willing to work hard or who have significant outside demands on their time that might tend to reduce their productivity or availability.

Often indivisibilities exist in both supply and demand for services, but are at least roughly congruent with each other. Dentists presumably prefer to provide complete dental procedures rather than partial ones, and
patients strongly concur—no one wants to buy just the "drilling out" portion of a cavity-filling procedure. In this context, lumpiness presents few problems, although the full lump may be financially unattainable for some patients. It is interesting, however, that the lumpiness is much more acute for the patient. A holdup problem might occur if dentists could perform the drilling-out portion and then renegotiate the price for the filling's completion. Luckily, professional norms, law, and repeat play protect consumers against this strategy, but we can see a similar problem in some other contexts.

For example, a leading actor who performs for an entire season of a television show or an entire run of a play may generate many times more value than if he appears for only part of the series. Even if the performer also gains a greater lump of value (in terms of fame or reputation) from completing the entire series than quitting midway through, he may have much less to lose from dropping out than the show’s producers do. What is to stop such an actor from threatening to walk off the project partway through unless the contract is renegotiated on more favorable terms? This is exactly what James Gandolfini, star of The Sopranos, did at one point (and it worked—he got more money).\(^5^3\)

More broadly, indivisibilities present the potential for contracting parties to apply leverage to each other. Renovations, auto repairs, medical procedures, and many similar services exhibit indivisibilities that make it difficult for consumers to readily switch to a competitor midway through. Information asymmetries may also make it difficult to know whether an announced change in price as the work progresses represents a strategic ploy to exploit the leverage provided by the lumpy situation or simply a response to new information that has been uncovered in the earlier phases of the work. In some contexts, dual sourcing or similar approaches can alleviate switching costs and potentially police strategic efforts to extract more surplus.\(^5^4\)

**Events and Conditions**

Many important outcomes have a lumpy or binary quality—a population of animals crashes or remains sustainable, a candidate is elected or defeated, an accident occurs or it does not. When investments made by different parties combine to produce outcomes, the problem has features that resemble those involving contributions to a step good such as a bridge. The key is to induce each party to contribute amounts that, when combined, will be just sufficient, but not excessive, to produce the result. In
the examples just given, the events and conditions feature the same lumpy demand patterns as we have observed with goods and services.

Inputs to these desirable or undesirable conditions may themselves be lumpy as well—a form of lumpiness in supply. Consider the goal of avoiding an accident. Some variables, like driving speed, are continuous, but others are all-or-nothing: a car either has antilock brakes or it doesn’t. Getting to the no-accident condition requires combining enough contributions to safety, but figuring out how to get there when some contributions are binary and others are incremental can be challenging. Similar issues arise in keeping pollution below particular thresholds, where some inputs (like adding a scrubber to a factory) are indivisible and others (like reducing operating hours) are incremental. In addition to finding the “cheapest cost avoider,” it may be important to identify who is the cheapest precaution slicer—the party best able to scale precautionary inputs to avoid a lumpy event like an accident.

**Personal Goals**

Often people set goals for themselves (or have goals set for them by others) that have a lumpy or all-or-nothing quality. People may create rules that bundle together all instances of a given type of behavior (such as not drinking or not eating meat), or they may come up with plans that help them realize lumpy personal goods (like a fitness target or writing a book) or avoid lumpy bads (such as alcoholism or other forms of addiction). The ability of people to achieve their goals may be heavily influenced by the way their choice sets are configured, which depends in turn on how markets and law interact. Lumpiness plays a large role in human cognition more generally. Indeed, many common aphorisms testify to the ubiquity of these considerations in everyday life, such as “in for a penny, in for a pound,” “it’s only a drop in the bucket,” “well begun is half done,” “it’s now or never,” or “it’s the least I could do.”

**Law**

Law interacts with many forms of lumpiness that have already been introduced. Perhaps most obviously, law can make it easier or harder to slice up unified things or assemble fragmented things. For example, eminent domain allows certain kinds of land aggregations to occur more easily, while other legal rules address the slicing up of unified property interests. There
are many laws and regulations that encourage or discourage, even when they do not mandate or forbid, particular ways of dividing up everything from risk to contractual obligations to families to jobs to units of housing. The law may also specify minimum or maximum lumps of production or consumption (such as minimum lot sizes or maximum soft drink sizes).

Moreover, law is often used to bring about or avoid circumstances, conditions, or occurrences that have a lumpy or step quality. The tax system, for example, mandates contributions that ensure that enough money will be aggregated to purchase lumpy public goods like bridges. Regulations operate to keep a fishery sustainable or to keep pollution below a critical threshold. Likewise, there may be a threshold level of enforcement of criminal laws that must be met within a given jurisdiction before inhabitants enjoy a sense of “law and order,” and a minimum level of property rights protection that is necessary to induce widespread investment and reliance. Uniform accessibility requirements like curb cuts or wheelchair ramps can enable mobility throughout an entire community, producing an aggregate value analogous to that of a completed highway. And even the mundane legal restriction of banning smoking in bars lets barhoppers dodge the lumpy bad of smelly clothing that even one smoke-filled bar would inflict.

Finally, law itself may exhibit lumpiness. Many legal outcomes are all-or-nothing—a defendant is guilty or not guilty, liable or not liable, required to hand over a disputed piece of property entirely or allowed to keep it forever. In making these binary choices, law must also decide how the process of choosing a winner will proceed, including how the inputs to particular legal outcomes—such as pieces of evidence—will be aggregated together or considered separately. When a driver suffers a lapse of attention, for example, should we look just at the fateful moment or at her larger pattern of driving behavior in assessing liability?

There may also be lumpiness in the supply of legal rules, if there are high fixed costs or other considerations that make producing additional laws or legal classifications costly. Consider *numerus clausus*—the notion that only a fixed, limited number of property forms are permissible and that further customization is disfavored. In Thomas Merrill and Henry Smith’s account, the limited number of forms economizes on information costs. People interacting with the property system may prefer that property interests be delivered in a small number of familiar forms, not only to make transacting easier, but also so that they can understand their own holdings and avoid encroaching on those of others. Likewise, regulations
may be easier to produce and understand when they cluster around a few standard property forms than if different laws must be created and heeded for an infinite variety of alternatives.\footnote{This book will answer that question in some detail. To preview, there are three main reasons we should care about lumpiness—and, by extension, about problems of segmentation and division.

First and most obviously, the concept of lumpiness bears on a wide range of efforts to optimally configure resources, from land assembly to car sharing. I show how problems of dividing and aggregating are not distinct problems, but rather share a common structure, one that is informed by attention to lumpy production functions.

Second, an understanding of lumpiness allows us to recast many collective action problems, legal puzzles, and social conflicts in terms of indivisibilities and complementarities, which makes it easier to resolve them. Many of the most difficult problems known to law and policy involve choosing between two (or more) sets of complementary goods, and lumpiness offers a framework for doing so.

Third, lumpiness can be intentionally leveraged to advance personal or social goals by altering or constructing the choice sets that actors confront. Interactions with others and even with oneself look different if moves can only be made in certain-sized chunks than if they can be selected in fine degrees from a continuous menu.

Through these channels, lumpiness influences private and informal governance regimes, formal law, and even the efforts of individuals to manage different temporal versions of themselves. Its significance extends from the most personal realms (an individual's efforts to complete a project or stick to a diet) to the largest and most public concerns (such as eminent domain, housing policy, or environmental protection). The balance of the book will show how lumpiness cashes out in a range of contexts.}

\subsection*{Why Should We Care?}

This survey of lumpiness might seem to prove too much. If so many everyday phenomena can be recast as lumpy or indivisible, we might wonder how significant the concept can really be. Why should it merit our attention? This book will answer that question in some detail. To preview, there are three main reasons we should care about lumpiness—and, by extension, about problems of segmentation and division.

First and most obviously, the concept of lumpiness bears on a wide range of efforts to optimally configure resources, from land assembly to car sharing. I show how problems of dividing and aggregating are not distinct problems, but rather share a common structure, one that is informed by attention to lumpy production functions.

Second, an understanding of lumpiness allows us to recast many collective action problems, legal puzzles, and social conflicts in terms of indivisibilities and complementarities, which makes it easier to resolve them. Many of the most difficult problems known to law and policy involve choosing between two (or more) sets of complementary goods, and lumpiness offers a framework for doing so.

Third, lumpiness can be intentionally leveraged to advance personal or social goals by altering or constructing the choice sets that actors confront. Interactions with others and even with oneself look different if moves can only be made in certain-sized chunks than if they can be selected in fine degrees from a continuous menu.

Through these channels, lumpiness influences private and informal governance regimes, formal law, and even the efforts of individuals to manage different temporal versions of themselves. Its significance extends from the most personal realms (an individual's efforts to complete a project or stick to a diet) to the largest and most public concerns (such as eminent domain, housing policy, or environmental protection). The balance of the book will show how lumpiness cashes out in a range of contexts.
NOTES

INTRODUCTION
2. In a world without transaction costs, these feats (and others) could be accomplished effortlessly, sans capes. See Coase, “Problem of Social Cost.”
5. See Sloan.
6. See Frank, Production Theory, 117. This is not to suggest that economists have wholly ignored indivisibilities. They haven’t: sophisticated treatments of the topic exist. But the economic analysis that features in most legal scholarship generally assumes linear relationships. There are exceptions, of course, some of which will be discussed in this book, but lumpiness remains underappreciated.
7. See, e.g., Frank, 117 (observing that “the tools of algebra and mathematical analysis usually fail to be of much use in analyzing the effects of indivisible commodities”); Bobzin, Indivisibilities, 1 (“Even advanced works on microeconomic theory . . . refrain from the consideration of indivisible goods and factors to provide a structure for the analysis where relatively simple mathematical methods can be applied.”).
8. See, e.g., Arrow and Hahn, General Competitive Analysis, 62.
9. Mas-Colell, “Non-Convexity,” 655. The horse and oat example is from Walras, Elements of Pure Economics, 95, quoted in Mas-Colell, 655. See also Frank, Production Theory, 117.

CHAPTER ONE
1. See Waldfogel, Tyranny of the Market (examining how fixed costs limit product availability); Mas-Colell, “Non-Convexity,” 656 (describing labor specialization as a response to indivisibilities in learning skills). Indeed, were it not for scale economies,
each of us could "assemble in our own backyards all of the manufactured goods whose services we would like to consume." Scarf, "Allocation of Resources," 114–15.


3. Wicksteed, Common Sense, 97-98.

4. For work analyzing Solomon's decision, see, e.g., Brams and Taylor, Fair Division, 6–7 and n2; R. Brooks, "Relative Burden," 282 and nn62–64.

5. See, e.g., Young, Equity, 13–14.

6. See, e.g., Benkler, "Sharing Nicely."


10. See Prabhat, “‘Borrow My Doggy.com.”

11. See Young, "Dividing the Indivisible," 904, 906; see also Frank, Production Theory, 32 (giving the example of "an industrial heat exchanger with a two-million-ton capacity," which if split, would comprise "two piles of steel scrap and other debris," not "two heat exchangers with a capacity of a million tons apiece").

12. See Frank, Production Theory, 32 (listing four different senses in which a commodity might be considered "indivisible" including "where a given amount of a commodity cannot be physically divided into fractional parts in any meaningful sense").

13. For a helpful discussion of production functions, see Oliver, Marwell, and Teixeira, "Theory of the Critical Mass."

14. See Oliver, Marwell, and Teixeira, 525–28 and fig. 1 (depicting and describing a variety of production functions).


19. See, e.g., Hampton, "Free-Rider Problems, 249–50 (discussing "steppy" collective goods, for which contributions in particular increments will add value, and "mixed structure" collective goods, which may require an initially large production step but could then be improved in smaller increments).


21. For graphical representations and analyses of possible land assembly scenarios, see, e.g., McDonald, "What Is Public Use?," 15–19; Fennell, “Taking Eminent Domain Apart,” 972–75.

22. Definitions of lumpiness vary in breadth. Compare Hampton, "Free-Rider Problems," 248–50 (equating "lumpy goods" with "pure step goods" and distinguishing both from hybrid forms like multistep and mixed goods) with Levi, Of Rule and Revenue, 57–58 (recognizing the possibility of "lumpy goods with sloping risers" that exhibit linearity "after the initial production threshold is crossed").


24. See Faden, Economics of Space and Time, 208, 213.

Notes to Pages 17–22 / 239

27. Bostedt, "Threatened Species." For discussion and additional examples, see Buchholz, Cornes, and Rübbelke, "Public Goods and Public Bads."
28. Bostedt's analysis is not framed in this way, but it does imply at least one form of lumpiness. Bostedt, "Threatened Species," 61 (citing surveys indicating a widespread preference for the existence of the wolf regardless of its numbers, which would be consistent with a sharp step at the level of species sustainability).
29. Smith, "Law of Things," 1693; see also Fennell, "Lumpy Property."
31. Such changes may reshape property expectations. See Nash and Stern, "Property Frames," 484.
32. See G. Alexander, "Objects of Art" (using examples from the work of artist Félix González-Torres that evolve with audience participation).
33. Van Inwagen, Material Beings, 104.
34. See Van Inwagen, 33–37.
35. Rogers and McAvoy, "Mule Deer Impede Pando's Recovery."
39. See chapter 11.
40. See, e.g., Adler, Well-Being and Fair Distribution, 405–75; Fennell and Stark, "Taxation over Time." Related philosophical questions surround the durability and cohesiveness of personal identity. See Parfit, Reasons and Persons.
41. This feature of nonrival goods enables increasing returns to scale that can fuel exponential economic growth, as more people make use of the good as an input to production. See Romer, "Endogenous Technological Change."
42. Some nonrival goods, like cleaning up a neighborhood or tidying a shared apartment, do not have this lumpy quality, assuming that greater and lesser degrees of cleanliness can be meaningfully enjoyed. See Frohlich and Oppenheimer, "With a Little Help," 109; Lunney, "Discrete Public Goods," 6–16.
47. See, e.g., Baumol and Sidak, "The Pig in the Python," 385; Spulber and Yoo, "Access to Networks," 913.
48. Waldfogel, Tyranny of the Market, 21–28, 100–107; see also Faden, Economics of Space and Time, 213.
49. See Anderson, Long Tail.
50. See Waldfogel, Tyranny of the Market, 134–38.
51. A caveat to this point will be discussed in chapter 7, where the nature of the work
is itself inherently lumpy. See Van Echtelt, Glebbeek, and Lindenberg, "New Lumpiness of Work."

52. See, e.g., Landers, Rebitzer, and Taylor, "Rat Race Redux."

53. This is one of several examples discussed in Shavell, "Contractual Holdup and Legal Intervention," 327–28. Gandolfini’s per episode pay was reportedly increased from an initial contractual level of $400,000 to over $800,000. See Reuters, "Sopranos Kingpin Set for Raise," March 18, 2003, http://www.cnn.com/2003/SHOWBIZ/TV/ 03/18/television.sopranos.reut/.

54. See Singer, "Competitive Public Contracts" (proposing "competitive dual sourcing" for public contracts).


56. See Weisbach, "Disability Law," 98.


58. See, e.g., Leo Katz, Why the Law Is So Perverse, 139–55.

59. See Nou and Stiglitz, "Regulatory Bundling," 1202–03 (discussing "rule-production costs").


61. See generally Merrill and Smith. This account has not gone unquestioned. See, e.g., Robinson, "Personal Property Servitudes," 1484–88.

62. See Davidson, "Standardization and Pluralism," 1601–3, 1644–50 (discussing limited property forms as "regulatory platforms").

CHAPTER TWO
2. For a detailed analysis of the Kelo decision and its aftermath, see Somin, Grasping Hand.
5. This trade-off between investment efficiency (getting people to optimally develop and maintain their property) and allocative efficiency (getting property into the hands of those who value it most highly) is well framed in Posner and Weyl, "Another Name for Monopoly."
6. See, e.g., Heller, "Tragedy of the Anticommons" (examining the effects of multiple necessary permits to open new storefront businesses in post-Soviet Russia); Chang and Fennell, "Partition and Revelation" (considering problems in the partition of land among co-owners).
7. See Kominers and Weyl, "Assembly of Complements"; see also Winn and McCarter, "Who’s Holding Out?,” 184–85 (finding in an experimental study that even weak competition, in the form of an imperfect substitute, was effective against seller holdout problems).
9. See Kominers and Weyl, 362.
10. On the difficulties presented by changes over time in the efficient scale of use, see, e.g., Bell and Parchomovsky, "Reconfiguring Property," 1024; Fennell, "Commons, Anticommons, Semicommons," 48.