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Quadratic Voting as Efficient Corporate Governance

Eric A. Posner & E. Glen Weyl

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Abstract. Shareholder voting is a weak and much-criticized mechanism for controlling managerial opportunism. Among other problems, shareholders are often too uninformed to vote wisely, and majority and supermajority rule permits large shareholders to exploit small shareholders. We propose a new voting system called Quadratic Voting (QV), according to which shareholders are not given voting rights but may purchase votes, with the price of votes being a quadratic function of the number of votes purchased. QV ensures that voting outcomes are efficient under reasonable conditions. We argue that corporations should implement QV, or a simple approximation called square-root voting, and that the law permits them to do so. Certain legal protections for shareholders, such as the appraisal remedy and poison pill, are unnecessary if QV is implemented.

Introduction

Since Berle and Means’ classic book of 1932, the agency costs of corporate governance have played a central role in discussions about corporate law. Berle and Means observed that in the modern publicly held corporation, shareholders cannot realistically control managers, which means that managers can take a range of actions that transfer the corporation’s wealth to themselves rather than to the shareholders. In modern terms, corporations are beset with agency problems. Large portions of corporate law can be understood as an attempt to minimize agency costs.

The central problem is that the managers of the corporation exercise control over its activities, and have inside information about which activities are profitable and which are not. Large corporations have thousands or millions of shareholders because shareholders seek to diversify their holdings, and so avoid buying all or nearly all of a firm’s shares. But by the same token, shareholders lack information about the workings of a corporation, and thus have trouble judging the managers’ decisions. As a result, managers can take actions that fail to maximize the value of the corporation and instead transfer value to the managers themselves.

Examples of such managerial opportunism are well known. At the extreme, managers can simply expropriate some of the firm’s assets. This is unusual in advanced countries, but managers can accomplish the same goal sub rosa by overpaying themselves, diverting corporate

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1 Kirkland & Ellis Distinguished Service Professor, University of Chicago Law School; Assistant Professor in the Department of Economics and the College, University of Chicago. Thanks to Todd Henderson and conference participants for comments, and Ellie Norton for research assistance.


3 See Andrei Shleifer and Robert W. Vishny, A Survey of Corporate Governance, 52 J Fin 737 (1997) for a valuable but somewhat dated survey. For recent discussions, see, for example, Jean Tirole, The Theory of Corporate Finance (Princeton 2005).
opportunities to independent entities that they control, overinvesting in perquisites like fancy office suites, building empires so as to enhance their sense of importance, and so on. Some commentators have blamed the financial crisis of 2007-2008 on poor corporate governance at major financial institutions.⁴

There are two direct mechanisms for controlling managers. First, shareholders enjoy voting rights with respect to major actions like mergers, elections of members of the board of directors, amendments to corporate charters, and stock issuances. Second, the law provides remedies when managers engage in the worst forms of self-dealing, like appropriating assets. Managers are also, of course, indirectly constrained by other factors, such as product market competition and the threat of takeover.

Our focus is on the voting system. The idea behind voting is that if shareholders can exercise the vote, they can block transactions that do not maximize shareholder value. But there is also an obvious problem with shareholder voting. A voter (or coalition of voters) with a majority of shares (and hence votes) can outvote the minority, and so cause the corporation to make decisions that transfer value from minority to majority, including decisions that do not maximize firm value. Since investors can anticipate such majority opportunism, they will pay less for equity than they otherwise would.

We propose a superior form of corporate voting known as Quadratic Voting (QV), which is based on theoretical work by Glen Weyl.⁵ Under QV, shareholders do not obtain voting rights along with their shares. Instead, everyone interested in a corporate outcome that is subject to a vote may buy as many votes as he wants for the purpose of casting them in that particular election. The price of the votes is a quadratic function of the number of votes purchased. For example, one can buy one vote for $1, two votes for $4, and three votes for $9. One can also buy fractions of votes, again for the square of the fraction. The proposal subject to the vote is approved if the number of votes in favor exceeds the number of votes against. The money collected from the voters is transferred to the corporate treasury, and thus ultimately distributed to the shareholders, except that large shareholders (with more than 1% of stock) would only receive back 1% of the money collected from the votes they personally buy. Any excess thus generated would be distributed pro rata by shares directly to the rest of the shareholders. The voting process is confidential and collusive arrangements and side payments would be illegal and subject to enforcement under antitrust law.

Under reasonable conditions, QV guarantees an efficient outcome, which reduces agency costs by preventing managers from implementing major decisions that benefit them at the expense of the firm, and preventing large shareholders from directing the corporation to enrich themselves at the expense of minority shareholders. There are also a number of positive second-order effects. QV increases the value of corporate votes, so that more will be held; and this further constrains managers and large shareholders, reducing agency costs. QV may improve the

⁴ See, for example, David H. Erkens, Mingyi Hung, and Pedro Matos, Corporate Governance in the 2007-2008 Financial Crisis, 18 J Corp Fin 389 (2012); Lynne L. Dallas, Short-Termism, the Financial Crisis, and Corporate Governance, 37 Iowa J Corp L 265 (2012).
incentive of investors to gather information about firms and to vote. Finally, as we will discuss, QV may render unnecessary certain legal protections of shareholders, such as the appraisal remedy and poison pills, which scholars have long regarded as costly and imperfect. For all these reasons, QV should lower the cost of capital.

I. Quadratic Voting

Most groups make collective decisions about public goods by some form of one-person-one-vote democracy, or one-share-one-vote in the corporate case. This contrasts sharply with the systems modern economies use to allocate private goods, where those who care more about a good receive more of it. Because typical majority-rule voting systems in democracies do not allow individuals to express intensities of preference, they are not efficient in creating public goods in the way that the market is efficient in allocating private goods. As a result, economists have proposed alternative social choice “mechanisms” that incorporate intensity of preference and provide individuals with an incentive to optimally disclose their true intensity of preference. Unfortunately, these mechanisms have thus far suffered from various severe defects that make them impractical. For example, the most canonical of these mechanisms, that proposed by Vickrey, Clarke, and Groves,\(^6\) can be turned to the purposes of any two individuals who collude in their participation in the mechanism or any one individual who can manufacture a second, false identity.\(^7\)

QV, as described above, avoids these difficulties while allowing intensity of preference to be expressed. To see why this rule incorporates the true intensity of individuals’ preferences, consider two different individuals. The first individual has a net value for building a bridge of $1000; that is, she would be willing to pay $1000 to see the bridge built. The second individual opposes construction of the bridge and is willing to pay $500 to avoid the taxes its building will impose on him. Both individuals think that the vote on the bridge is likely to be close enough so that each vote they purchase moves the chance that the bridge is built or not in their desired direction by 1%. Because both are rational and neither has a very large impact on the decision, they share the same value of this estimate and it is independent of the number of votes they buy so long as this number is not too large.

How many votes will the first individual buy in favor of the bridge being built? Each vote gains her a value of \(0.01 \times 1000 = 10\). If she buys only one vote, she spends $1 and receives a benefit of $10. If she buys 2 votes, she spends $4, and so for a marginal cost of $3 ($4-$1), she gains another $10. What about 3 votes? Three votes cost her another $5 ($9-$4), still less than the marginal gain of $10. If she buys 4 votes, she pays $16, while if she buys five votes she pays $25, so it is better for her to buy 5 votes rather than 4—because the cost of the additional voting power ($9) is less than the gain ($10). If she buys 6 votes she pays $36, $11 more than if she buys 5 votes, but only gains $10 in value from doing so. Thus she won’t buy the sixth vote.


Using a little bit of calculus we can solve for her optimal number of votes \( v \). The value she gains, in dollars, from buying votes is \( 10v \) while the amount she loses from the cost of votes is \( v^2 \). Setting her marginal cost equal to her marginal benefit of buying votes, her optimal number of votes is \( 10 = 2v \) or \( v^* = 5 \). So she should buy exactly 5 votes.

On the other hand, the individual who is willing to pay \$500 to avoid the bridge being built will buy negative votes (votes against the bridge). His utility from votes is \( -5v - v^2 \). By the same logic, he will optimally buy \( v^* = -2.5 \). More generally, an individual with utility \( u \) from the bridge buys votes \( v^*(u) = \frac{0.01u}{2} \), because the chance that she changes the outcome is 0.01. The decision is made based on total votes, which are the same as the total value of \( \frac{0.01}{2}u \). But this is positive if and only if the sum of everyone’s utilities is as well. That is, the system will make a decision in favor of whichever outcome (building or not building the bridge) maximizes the total utility because everyone buys votes in proportion to the intensity of their preference. QV is thus efficient, as long as the approximating assumptions we used above hold.

In particular, Weyl shows that as long as the distribution of preferences is commonly known and some technical assumptions are satisfied, the probability that the decision is made incorrectly approaches 0 as the number of voters \( n \) grows large at rate \( \frac{1}{\sqrt{n}} \). If the distribution of preferences is not commonly known and individuals must make guesses about it based on their own preferences or the total number of voters is small, matters are more subtle. However, QV is still fully efficient under reasonable conditions, and much more efficient than voting under a broad range of cases.

What is so special about the quadratic rule? Why couldn’t individuals just pay proportional to the number of votes they buy? The key is that the quadratic rule is the only one under which the cost of the marginal vote is proportional to the number of votes already bought. If, as we assume, the marginal benefit of a vote increases at a linear rate, then the marginal cost of the vote must also increase at a linear rate; that is only possible if the total cost of votes increases by the square of the number of votes.

By way of contrast, suppose that votes had a fixed linear cost of, say, \$7. Then the first individual would be willing to buy an enormous number of votes, as each is worth \$10 to her and the second individual would be willing to buy none. The first individual would thus act as a dictator, buying the whole election even if there were 10 or 100 other individuals all of whom would be willing to pay \$500 to avoid the bridge being built. It is this disproportionate power afforded to those who are willing to pay most that makes standard linear vote buying so unattractive and is likely the reasons so many are opposed to corruption of politicians, “empty voting” for corporate governance (where individuals buy votes without having to buy shares using derivative contracts), and other cases where an individual can linearly buy influence in a

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8 The rate depends on the tail properties of the distribution of valuations and the result reported here is based on the conservative assumption of a Pareto-tail coefficient of \( \alpha = 3 \). If tails of the valuation distribution are thinner than this, then the rate of convergence is faster. Weyl, *Quadratic Vote Buying* (cited in note 5).

collective decision. As we discuss in the next subsection, this problem can be severe even if votes are linked to shares.

In practice, though, individuals who feel intensely about an issue will try to use their financial resources to influence the outcome, usually by spending on persuading the population to vote their way. In our democratic system, such spending is constitutionally protected. While such expenditures may have some of the decreasing returns to expenditures embodied in QV and thus may roughly approximate efficiency, there are many reasons to think an explicit vote-buying scheme would work better. First, by formalizing the exactly correct rule, QV enhances efficiency and provides a procedure that is simpler and more reliable for participants than the status quo is. Just as the introduction of fiat money, accounting and formal contract law made informal systems of reciprocity and exchange that prevailed prior to the modern era into a widespread and efficient market economy, QV could turn the somewhat chaotic system of bidding for influence more systematically efficient. Second, expenditures on persuasion under the current system are largely wasted, while the revenue raised on QV would be distributed back to the population or spent on valuable projects. Finally, QV would discourage further expenditures on persuasion intended to push nearly-indifferent voters slightly one way or the other. Because votes are costly and the system incorporates preference intensity, persuasion would aim to convince everyone that a policy or candidate would increase their utility more than the alternative rather than just breaking indifferences for “swing” voters.

Vote buying seems to carry negative connotations, but it is crucial to notice the difference between QV and standard, linear vote-buying. For one thing, the buyer does not pay a particular voter (or shareholder) but society (or the corporation), so the payments are spread among all voters rather than concentrated in a few. But, more important, the quadratic formula blunts the impact of money and hence wealth on outcomes. If Mitt Romney had wanted to spend nearly his whole personal fortune, $100 million, to win the presidential election, that would be his right…but he could only buy himself 10,000 votes, a substantial number but hardly enough to guarantee victory given that President Obama’s margin was nearly 5 million votes. And at the same time, the money he spent could be used to pay off the deficit, fund government programs or cut taxes rather than being wasted on advertisements.

An important concern with other mechanisms economists have proposed was that they allowed for easy and severe collusion: any two individuals willing to collude, or any individual passing himself off as two individuals, could get whatever outcome they wish and pay nothing. Collusion is also possible under QV, but is much less serious. Consider again the example of Mitt Romney. If he were able to collude with 99 other people, or to take on 99 other false identities, he could divide his $100 million into 100 groups of $1 million and purchase 100,000 votes instead. This is much more than he would be able to do on his own. However, he would run into two obstacles. First, 100,000 votes still is not enough to swing many large elections. Second, especially if a secret ballot were enforced and fraud illegal, it would be hard to maintain such collusion. His collusive partners would have a strong incentive to pocket the money and not buy the votes and if he tried to pass himself off as 100 different people he would likely be caught. Thus, because under QV individuals need to collude with or pretend to be a large number of other individuals to make a significant difference in the amount of influence they can exert, its

sensitivity to collusion is not excessive. Weyl makes these arguments quantitatively precise and formal.\textsuperscript{11} We thus believe that QV’s benefits in terms of allowing individuals to express the intensity of their preference outweigh its costs in terms of collusion or fraud. In the corporate context, where contract and antitrust laws can be used to deter collusion and First Amendment rights to political spending are not at issue, the problem of collusion is even more limited.

II. QV and Corporate Governance

A. Corporate Voting and Its Pathologies

A publicly traded corporation is normally operated by its management, under the loose supervision of the board of directors. Shareholders have no say in day-to-day decisions. When a firm enters a “major” transaction, however, management must seek the approval of shareholders, and sometimes other stakeholders such as creditors. Major transactions include mergers, certain large sales of assets, financial transactions like stock issuance that could dilute the value of existing shares, and bankruptcy reorganization plans (where creditors can vote as well). We will focus on these types of transactions, and use a merger as our running example.\textsuperscript{12} But shareholders can vote in other contexts as well, including in elections of members of the board of directors and on amendments to the corporation’s charter.

The voting rules are largely determined in the charter, but they are subject to certain (relatively limited) legal requirements. Votes are cast by share, not by shareholder; a shareholder who owns 100 shares has 100 votes. Shareholders can thus accumulate votes by buying additional shares. Voting is by majority or supermajority rule. For example, approval of a merger requires a majority of outstanding shares in most U.S. states; various supermajority rules are used in some other countries.\textsuperscript{13} Minority shareholders can thus be outvoted.

The weakness of the voting rules, as well as other corporate governance mechanisms, is the dominant theme of the corporate government literature. Scholars focus on two basic problems. The first is managerial opportunism, where managers implement projects that transfer wealth from shareholders to managers. Such projects could include outright appropriation, excessive executive compensation, the diversion of corporate opportunities to managers, overinvestment in management perquisites, and managerial entrenchment.\textsuperscript{14}

\textsuperscript{11} Weyl, \textit{Quadratic Vote Buying} (cited in note 5).
\textsuperscript{12} A small literature discusses mechanism design for mergers and acquisitions. See, for example, Steven J. Brams & Joshua R. Mitts, Mechanism Design in M&A Auctions (2013), available at Peter Cramton & Alan Schwartz, \textit{Using Auction Theory to Inform Takeover Regulation}, 7 J. L. Econ. & Org. 27 (1991); Alexander Gorbenko & Andrey Malenko, \textit{Strategic and Financial Bidders in Takeover Auctions} (2009), available at http://www2.lse.ac.uk/fmg/documents/events/seminars/capitalMarket/2010/1310_Gorbenko%20.pdf. These papers discuss the process that the board of directors should be required to use when soliciting bids; by contrast, our focus is on the process that shareholder use to constrain the board of directors when it comes time to approve the merger or acquisition.
\textsuperscript{14} Shleifer and Vishny, 52 J Fin at 742–43 (cited in note 3).
The second problem is that of tyranny of the majority, where majority shareholders use their voting strength to expropriate value from minority shareholders. Again, the way that majority shareholders use their voting power will vary. For example, they could (in principle) vote for outright appropriation of the shares of the minority. But more typically, they use their voting power in more indirect ways—for example, approving mergers and other transactions that benefit the majority at the expense of the minority, issuing shares that dilute minority interests, and so forth.\textsuperscript{15}

The voting system thus addresses managerial opportunism by giving shareholders the power to block value-reducing transactions, but creates the problem of tyranny of the majority by giving the majority the power to outvote the minority. A supermajority rule can reduce but not eliminate the risk of value-reducing transfers from a (smaller) minority to a (larger) majority, but also creates the problem of hold-outs by a small group who withhold approval until paid off. These additional transaction costs may block efficient transactions.\textsuperscript{16} Thus, ordinary voting fails in its fundamental function of aggregating beliefs and preferences of diverse shareholders to ensure efficient corporate governance.

Even where the system works in theory, managers and majority shareholders can often evade the rules. For example, a corporation may avoid a vote on a merger by creating a shell corporation that buys the target firm.\textsuperscript{17} Or a corporation can effectively eliminate the value of voting in director elections by failing to provide shareholders with information about candidates who seek to challenge board choices.

In corporate bankruptcy, the pathologies of the voting rules are also evident. Shareholders cannot be trusted to vote so as to maximize firm value because they do not have residual ownership rights in an insolvent firm. The law gives voting rights to creditors. But different groups of creditors may have different interests, and so the law creates a complicated classification system, requiring majority (by number of class members) and supermajority (by amount of claims) voting by class, and approval of all classes subject to a cram-down exception. The bankruptcy court must also ensure that stakeholders are treated fairly and equally. There is very little reason to believe that these rules maximize the value of firms or treat creditors or anyone else fairly.\textsuperscript{18}

B. The QV Approach

A corporation could implement QV by amending its charter. The charter would provide that all shareholder voting would be conducted through QV. In particular, the charter would provide that whenever a board election, charter amendment, or major transaction occurs, a QV election would take place. Anyone—including people with no relation to the corporation—would have the right to participate. Note that shareholders would \textit{not} vote the number of shares they

\textsuperscript{15} Id at 758–59.
\textsuperscript{17} See Stephen M. Bainbridge, \textit{Corporate Law} 348–49 (Foundation 2d ed 2008).
own; large shareholders and small shareholders are in the same position as each other, and indeed in the same position as non-shareholders, who would also have the right to participate. The funds paid in from QV would be disbursed into the corporation’s treasury and thus would disproportionately benefit large shareholders and would not benefit non-shareholders. However, no shareholder would ever be able to receive back (implicitly or explicitly) more than 1% of the funds raised by the votes she bought herself. If a shareholder has a greater than 1% shareholding, any funds she contributes would be distributed back directly to shareholders, with 1% going to her and the remaining funds being directly rebated to other shareholders in proportion to their holdings pro rata, even if these other shareholders are themselves large. This rule prevents the implicit price of votes from being (more than slightly) lower for a large voter compared to a small voter.

QV is superior to majority- or supermajority-rule voting because it ensures ex-post efficient outcomes. QV blocks managerial opportunism because managerial opportunism is by definition inefficient. And QV minimizes the tyranny of majority shareholders by preventing majorities from using inefficient projects to transfer value from minorities to themselves.¹⁹ When the majority supports an inefficient merger, the minority will outvote it by buying a larger number of votes. Because the minority loses more than the majority gains, and QV guarantees ex-post efficiency, QV will block the transaction. When the majority gains more than the minority loses, the transaction will take place, but it is ex-post efficient, and the minority will be protected in part by its payoffs under QV.

Of course, ex-post efficiency is not the only, or even the primary, goal in designing a corporate charter. Instead, charter-writers seek to maximize the amount of capital raised, which depends on the marginal value of owning additional shares ex-ante, in anticipation of future decisions, rather than total efficiency ex-post, as we discuss in Appendix A. While there is no guarantee QV will maximize this ex-ante shareholder value, it seems unlikely any mechanism could, given that individuals’ incentives ex-post will always be based on how the decision affects their level of ex-post utility rather than their ex-ante marginal willingness-to-pay for shares. Normal voting, for example, seems far less likely to maximize ex-ante shareholder value, especially given that the inefficient manipulations by majority shareholders to expropriate minority shareholders (impossible under QV) are a sure-fire way to destroy ex-ante value of shares. Maximizing ex-post efficiency seems much more likely to align with ex-ante shareholder value than do other equally ex-post but also inefficient rules.

The most unusual feature of QV from the standpoint of corporate law is that anyone—including non-shareholders—can vote. Thus, large shareholders have no greater voting power than small shareholders, and shareholders have no greater voting power than creditors, employees, neighbors, or even ordinary people with no relationship with the corporation. If this feature of QV seems puzzling, note first of all that most people in the world will not exercise their right to vote for the simple reason that they must pay in order to vote, and they are extremely unlikely to out-pay, and hence out-vote, shareholders and others who have a pecuniary stake in the corporation. This is especially true given the fixed transactions costs of participating in such a vote. Similarly, large shareholders will normally pay more to vote than small

¹⁹ QV fails only when the population of voters is small or there is aggregate uncertainty about the distribution of valuations, and as noted above it outperforms voting typically in these cases.
shareholders will because large shareholders gain more from the outcome. Moreover, while non-stakeholders can vote, they do not receive any money from the vote purchases of others, as that money goes into the corporate treasury. Finally, bear in mind that even under the current system non-stakeholders can vote merely by taking the (usually) financially trivial step of purchasing a share of stock. The distinction between corporate stakeholders and outsiders has never been very clear.\footnote{When a corporation such as a mutual or investment fund serves as a common agent for multiple principals, the common agent can buy votes only as one individual. This means, for example, that if the corporation seeks to cast 10 votes, it must pay $100, rather than (say) $10 on behalf of 10 principals who each vote once for $1. This is true even if the funds used to purchase the votes belong to the principals rather than the agent. If principals delegate this decision to the agent, they would not then be allowed to vote a second time themselves, but would be able to monitor the behavior of the agent. This offers a simple means of extending QV to representative and other delegated situations: each individual can vote only under one identity, even when she represents many individuals. By monitoring their agent, these individuals forfeit the right to vote themselves. This leads to efficiency by precisely the same logic that makes QV efficient more generally. So long as the agent acts in the collective interests of the principals, as the votes she will buy on behalf of her principals will be proportional to the sum of their utilities.}

The main point to keep in mind is that QV guarantees that the transaction will be ex-post efficient, thus maximizing the value of the corporation. In this way, QV ensures that shareholder value is maximized. Regular voting rules like majority and supermajority do not ensure ex-post efficient outcomes, and so are unlikely to have superior incentives on ex ante behavior.\footnote{We will discuss one possible limitation on this argument in Part D below.} Under normal voting, in the absence of legal protections, outsiders with interests contrary to those of the firm may buy up a majority of shares and inefficiently undermine the firm. Under QV such behavior could only occur when it is socially efficient, which would greatly limit the scope for outside manipulation.

As a result of this feature, QV can be used more often than shareholder voting in the current system. Because QV is more accurate and robust than the current system, it would be reasonable to use it for transactions whose magnitude fall below the size that currently requires a vote. For this reason, advocates of shareholder democracy should endorse QV.

A recurrent criticism of the current system of shareholder voting is that shareholders have insufficient incentive to inform themselves because part of the benefit of their voting is externalized on other shareholders. Because they do not adequately inform themselves, they either do not vote or vote badly. Because of the close connection between the Vickrey-Clarke-Groves mechanism and QV, the results of Bergemann and Välimäki imply that under QV, unlike under standard voting, individuals have an incentive to optimally collect information that is relevant to their preferences and beliefs so long as these do not have spillovers to what decision is in the interests of others.\footnote{Dirk Bergemann and Juuso Välimäki, \textit{Information Acquisition and Efficient Mechanism Design}, 70 Econometrica 1007 (2002).} Obviously in the corporate context, much information does spill over to others as each small shareholder captures only a small part of the value of such information. QV will not provide efficient incentives for information acquisition in this case. Nonetheless, we conjecture, though we have not tried to prove, that QV is superior to traditional voting along this dimension. Intuitively, under voting, one only needs enough information to decide which side one favors. Under QV, one has an incentive to achieve a much finer
determination of the intensity of ones’ preference, which seems likely to increase information acquisition.

More generally, it is well-known that markets do not generally provide efficient incentives for information acquisition. As Hirshleifer argues, information is often under-provided because of the inability of the informed individual to appropriate the total social value of the information; it may also be over-provided when it is used, for example, to beat another individual to an arbitrage opportunity as is common in high-speed trading. Matters are very similar under QV: information relevant to the overall direction of preferences of many individuals will be under-provided, while information about how close the election is (and thus how valuable votes are) will be over-provided as this information is essentially zero-sum. Thus QV provides all of the benefits, and costs, of a market economy in its incentives for information acquisition, a set of trade-offs that, at least since Hayek, has been considered overall superior to those arising from democratic voting.

One might object that under QV people will refrain from buying large blocks of shares because they do not obtain voting advantages. People with large blocks of shares will have more intense preferences regarding mergers and the like, and so they will anticipate that they will spend more money on voting than people with small blocks of shares. But in the current system, the right to vote is close to worthless.\footnote{Henry T.C. Hu and Bernard Black, \textit{Hedge Funds, Insiders, and the Decoupling of Economic and Voting Ownership: Empty Voting and Hidden (Morphable) Ownership}, 13 J Corp Fin 343 (2007)} People typically buy large blocks of shares not to obtain voting rights but to obtain a larger portion of the profit of a firm. QV, by guaranteeing that the firm will enter only ex-post efficient transactions, should increase rather than reduce the value of equity. Furthermore, the funds raised through the expenditures on votes are added to the value of the corporation, raising the value of shares. It is plausible that this additional revenue will more than outweigh any increased value of shares from accompanying voting rights. For example, in recent experimental work on a voting system closely related to QV, Goeree and Zhang show that the revenue raised makes the overwhelming majority of individuals, even those who end up with much less influence, support QV over voting in experiments.\footnote{Jacob K. Goeree and Jingjing Zhang, \textit{Electoral Engineering: One Man, One Vote Bid} (Aug 27, 2012), online at \url{http://www.bm.ust.hk/ecdepts/Inefficient-Voting.pdf} (visited May 3, 2013).} We discuss these results in greater detail in Appendix A.

Indeed, a major problem with the current system of shareholder voting is that it compels corporations to inefficiently bundle the right to receive dividends and the right to vote. The two rights need not go together, as is clear from various preferred stock arrangements where voting rights are severed or diluted. It is not entirely clear why the law prohibits people from selling their votes independently of their shares. As we discuss in Part C, the best argument for the ban on vote-buying is that the current legal system requires majority or supermajority voting, and it is easier to circumvent these rules by buying votes alone (given that shareholders without controlling blocks value votes very little) than by buying shares.\footnote{For discussions, see Saul Levmore, \textit{Voting With Intensity}, 53 Stan L Rev 111 (2000); Henry T.C. Hu and Bernard Black, \textit{The New Vote Buying: Empty Voting and Hidden (Morphable) Ownership}, 79 S Cal L Rev 811 (2006).} A more fundamental reason is that linear vote buying would, as described in Section I, lead to the inefficient dictatorship of individuals with intense preferences. But this is precisely the problem solved by QV.

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Another possible objection to QV is that it would allow interest groups or others to shut down a corporation they dislike on ideological grounds. If they can buy votes without buying shares, they can destroy a corporation in which they have no economic interest. The problem with this argument is that these outsiders would have to spend an enormous amount of money to buy votes, and this money would go to outvoted shareholders. The transaction is only possible if the social value of the firm taking account of outsider interests is less than zero. Given that it would be in fact cheaper under the current system for outsiders to gain control of and shut down a firm simply by buying a majority of shares, and we virtually never observe such transactions, we are skeptical of this objection. Furthermore, as we discuss in our critique of appraisal remedies in the next subsection, unless such a group of outsiders is very large, under QV it is cheaper for them to pay the value of the firm plus a small sweetener to current shareholders so that shareholders will vote in favor of the proposal themselves than attempt to outvote the outsiders directly. Such a prospect can only increase value to shareholders. Corporate by-laws should ensure they do so by prohibiting side-payments contingent on the vote, forcing outsiders to pay only shareholders (and not a large group of outsiders) to persuade them to approve their plans. Enforcement of such rules could fall under antitrust law.

QV is not a panacea. It is subject to the same arbitrage risks as majority and supermajority voting. For example, if a board wants to avoid a vote on a merger, it still retains the option to undertake the merger through a shell subsidiary.\(^{26}\) However, at a minimum, these arbitrage risks are no worse under QV. And if QV is a better system, then shareholders may resist arbitrage (for example, by barring arbitrage transactions in the charter) because they care more about making their preferences felt through voting than they do under the regular system.\(^{27}\)

C. Square-Root Voting

Reforms of all types, and not just reforms to corporate law, often encounter a hostile reception simply because they are unfamiliar, and are at variance with entrenched norms. QV may face a similar fate because of some of its unusual features. Even though money plays a significant role in shareholder voting—rich shareholders who buy more shares vote more—there may be some uneasiness with explicit vote-buying, as one can see from the literature and cases on that topic.\(^{28}\) Allowing non-shareholders to vote also seems unusual. And the redistribution of funds collected through the QV process would be novel in the corporate context.

To address these concerns, we propose as a slightly more modest alternative variant of QV that we will call Square-Root Voting (SRV). SRV provides simply that only shareholders vote, and that shareholders have the right to vote the square root of the number of shares they

\(^{26}\) We have also not address how board voting would work; our focus instead has been on shareholder approval of transactions. Very roughly, board voting could work like this. SRV should be used to elect the directors. Shareholders vote in favor or against various candidates for the board and those who receive the largest net number of positive votes are elected to the board. Each director is then given a number of tokens equal to the number of net positive votes she received. The director then allocates her tokens across various QV votes over a given period of time. We do not have the space to defend this proposal here, and save it for future research.

\(^{27}\) On the other hand, managers set on opportunism may work harder to engage in arbitrage.

\(^{28}\) See below, section D.
SRV is formally almost identical to QV. To see why, consider the perspective of an investor who does not yet own any stock in Corporation X. Corporation X announces its intention to merge with Corporation Y. Under current law, the investor can purchase stock in X (or Y) and then exercise the voting rights associated with the share (or shares) of stock. Thus, the investor might buy the stock both because she believes that the merger will take place and that it will increase the value of the stock, and because she hopes to use the voting power to increase the probability that the merger will take place.

Under QV, the investor chooses the number of shares that she buys solely for the purpose of optimizing payment streams. She separately buys as many votes as is optimal for her to influence the outcome. Under SRV, the investor effectively buys stocks for their voting power. Because she can expect to resell them for the same price that she buys them for, she incurs only the opportunity cost of buying them. SRV ensures that this opportunity cost increases at a quadratic rate, and in this way resembles QV. If the ban on “empty voting” (trading votes and shares separately, as discussed in the next subsection) were dropped, SRV would be even closer to QV, differing only in the fixed number of potential votes.  

Yet SRV differs from QV in several ways that may be important as a matter of symbolism political rhetoric but are of little importance substantively. Most important, under SRV shareholders vote as they normally do; the only difference is that their voting power increases at a decreasing rate as the number of votes they cast increases. This approach differs in degree but not in kind from voting rules established in the charters of many corporations, for example, through the creation of classes with different voting rights, as are used for poison pills. SRV also does not permit non-shareholders to vote, and does not require the collection and the redistribution of funds. These similarities to existing practices may make SRV more palatable to judges and corporate boards than is QV, at least in the short-term, and may smooth the way for the adoption of more explicit QV in the long-term.

D. Legal Issues

As noted earlier, corporate law gives corporations a great deal of freedom to fashion voting systems. Although most corporations use regular majority or supermajority rule for major transactions like mergers, charter amendments, and board elections, there is a great deal of variation. For example, through a series of agreements with shareholders of Facebook, Mark Zuckerberg has 57.1 percent of voting control even though his economic stake in Facebook is only 28.4 percent.  

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29 This fixed supply is irrelevant as it only determines, by the laws of supply and demand, endogenously the multiplier in front of the quadratic term in the price of votes. Because this multiplier is irrelevant to all of the properties of QV it makes no difference.
A corporation could provide in its charter that shareholders would have the conventional right to a share of the firm’s profits, but that there would be no right to vote attached to each share. Instead, whenever the firm holds a vote, anyone registered as a shareholder at the time of the vote would have the right to participate. Each shareholder, regardless of how many shares he owns, would have the same voting power: the right to cast one vote for $1, two votes for $4, and so on. The payments would be made to the corporation, which would either deposit them in its treasury or put them in a fund, which would then be distributed pro rata to the shareholders at the conclusion of voting.

The major legal obstacle to QV is the somewhat qualified ban on vote-buying, but the law on vote-buying does not directly apply to QV. Indeed, existing law is better interpreted as directed at vote-selling—by shareholders—not vote-buying in the QV system, where the corporation, not a third party, sells the vote and the consideration is deposited in the corporate treasury. Thus, the traditional per se rule against vote-selling does not apply to QV on its terms.

Moreover, the rationale for that per se rule was that each stockholder should be entitled to rely upon the independent judgment of his fellow stockholders…. The apparent rationale is that by requiring each shareholder to exercise his individual judgment as to all matters presented, “[t]he security of the small stockholders is found in the natural disposition of each stockholder to promote the best interests of all, in order to promote his individual interests.”

Even if this rationale were accepted, it would not provide an objection to QV, because QV in fact advances the interest of small shareholders much more effectively than ordinary voting (with or without vote-buying) does.

Delaware courts now reject any per se rule against vote buying, and instead ban vote-buying when it is fraudulent or violates a test of “intrinsic fairness.” Clearly, there is nothing fraudulent about QV. The intrinsic fairness test is rather obscure, but we also see nothing unfair about QV since it ensures ex-post efficiency and, as we discuss below, will usually ensure compensation for outvoted minorities.

A number of academics have launched a separate line of attack on vote-buying. A controversy erupted a few years ago over “empty voting,” where parties engage in financial engineering in order to separate the vote and the economic value of the share it is attached to. For example, the owner of a share lends the share to an investor for a very brief period of time during which a corporate vote is held. The investor exercises the vote during this period but does not bear any economic consequences of the vote since the impact of the vote on the value of the firm takes place after the share is returned to the owner. An investor can also buy a share in order to obtain the voting right while fully hedging against any change in the value of the share.

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31 As we noted earlier, voting rights need not be limited to shareholders; but we suspect that in the early stages of its adoption, corporations are likely to confine voting rights to shareholders.
32 Schreiber v Carney, 447 A2d 17, 24 (Del Ch 1982), quoting Cone v Russell, 21 A 847, 849 (NJ Eq 1891).
33 Schreiber, 447 A2d at 25–26; Crown Emak Partners v Emak, 992 A 2d 377 (Del Ch 2010).
Traditional, linear vote-buying, whether in this form or another form, usually leads to inefficient outcomes. But these outcomes result only when vote-buying takes place in a regular voting system and thus, crucially, is linear. In fact, precisely the same analysis that Weyl uses to show the efficiency of QV implies that linear vote buying is inefficient. A broad class of vote buying rules nests democracy, dictatorship and QV all as special cases. Thus vote-buying in general, and QV in particular, should not be tarnished with the same brush as is linear vote-buying and its dictatorial results. As discussed in the previous section, “empty voting” and vote buying combined with appropriate rules (square-root voting) leads to a fair and efficient outcome.

Indeed, even if QV were deemed illegal, SRV would almost certainly survive a legal attack. Under SRV, the shareholder does not explicitly buy votes; she merely exercise the right to vote that accompanies her share. The fact that she can only possesses votes equal to the square root of the number of shares does not, even indirectly, implicate vote buying.

QV (or SRV) is not only lawful; it also reduces the need for corporate law that protects shareholders from managerial opportunism and minority shareholders from large-shareholder opportunism. Because QV blocks such opportunism, the protections are not necessary.

The most important such protection is the appraisal remedy. If majority shareholders engineer a merger that appears to offer unfair payoffs to the minority, the minority can seek protection in court by demanding an appraisal remedy. If it is successful, the corporation must pay minority shareholders the actual value of the shares, as determined by a court after a lengthy and complex valuation proceeding.

In a QV system, the ability to freeze out minority shareholders would be greatly diminished or even eliminated. There are two reasons for this. First, because QV blocks inefficient mergers, the majority would be unable to effect inefficient mergers in order to transfer value to itself. The majority would be able to engage in such opportunistic behavior only when the merger is efficient. Second and more important, QV is particularly unfavorable when a single shareholder or small group of shareholders attempt to impose their will on a majority. In Appendix B we show that, because of the small numbers, QV is not efficient in this case: it is biased against the small manipulative group! In particular, no individual or small group of shareholders can ever profitably succeed in outvoting other shareholders with any significant probability when the number of other shareholder-voters is large. Even when they do (with very small probability) win such a vote, the payments they make will typically fully compensate other shareholders for their loss. Thus, a shareholder or small group of shareholders seeking to execute such a plan would find it cheaper and more reliable to pay the shareholders enough of the surplus of such a transaction to make them want to vote for the transaction themselves. Intuitively, because the costs of voting are quadratic, and so the cost of casting the marginal vote increases at

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36 Weyl, supra note __.
greater than linear rate, it is cheaper as a small group to incentivize others to vote in your favor than it is to attempt to outvote them yourself. This self-interest provides a far more effective check on opportunism than does the bureaucratic and inaccurate appraisal process. The only case when appraisal could do better is when information is hidden from shareholders and is revealed through the appraisal.

A related and promising application of QV is in the area of poison pills. In response to a wave of hostile takeovers in the 1980s, many corporations amended their bylaws so as to make it more difficult for outside investors to succeed in their takeover efforts. A typical poison pill provides that when an outside investor obtains shares above a threshold (typically, a significant minority), then insiders may purchase additional shares at a discount (for example, 50 percent). Thus, insiders can buy up shares in order to obtain votes to oppose the takeover. Managers defend poison pills, claiming that takeovers frequently destroy jobs and disrupt operations. Shareholder activists argue that poison pills entrench management so that it does not pay for its mistakes by losing control of the corporation.

Poison pills are controversial but generally lawful in the United States, although not in all countries, and even in the United States, they can be challenged under general principles of corporate law. Now consider a poison pill that incorporates QV or SRV. A corporation could amend its bylaws to provide that any hostile takeover be subject to a vote under SRV. SRV does away with the threshold requirement (which is arbitrary) and replaces the simple discount rule with the more fine-grained quadratic function, which effectively makes it cheap for dispersed shareholders to vote for or against the merger but costly for large shareholders including management to oppose it. Since SRV guarantees ex post efficient results, it should ensure that takeovers are approved only when they are efficient. Poison pills should be lawful and immune to challenge as long as they comply with these principles.

Conclusion

QV holds great promise as a mechanism for eliciting people’s private valuations. It is well-suited to the corporate context, which lacks the norm of one-person-one-vote and where law and tradition permit people to use money to signal the intensity of their interests in managerial decisions. As with any innovation, QV merits experimentation prior to widespread adoption to help reveal potential weaknesses that eluded our analysis given the novelty of QV compared to the many centuries of experience with voting. The flexibility of corporate charter law allows for such small-scale experimentation by innovative, early-adopting firms that could then be studied by others and, if successful, spread. In the longer term, if such experimentation is successful we believe that QV could be useful in other settings. Kominers and Weyl advocate its use as an

37 See News Corp.’s poison pill. See []
alternative to eminent domain in land assembly;\textsuperscript{39} Posner and Weyl incorporate it into a proposal for reforming Chapter 11;\textsuperscript{40} and Weyl suggests variants that might be plausible for committee or broader public decision-making even if an aversion to “money in politics” persists.\textsuperscript{41}

\textsuperscript{40} Eric A. Posner & E. Glen Weyl, Voting Rules in Chapter 11: A Proposal for Reform, unpub. m.s. 2013.
\textsuperscript{41} Weyl, *Quadratic Vote Buying* (cited in note 5).
Appendix A: Ex-Ante Shareholder Value and Ex-Post Decisions

QV is ex-post (Kaldor-Hicks) efficient in the sense that, at the moment of decision, it maximizes the prospective wealth of society. The goal of a firm about to make an initial public offering, however, is not to maximize this ex-post efficiency but to minimize the cost of raising capital. To compare these two we consider a simple model.

There are two stages. At the first stage, a market for shares exists. At the second stage, a decision is made. There are $N$ individuals with a representative individual $i$ having utility $u_i^A(s_i) - ps_i$ if she owns $s_i$ shares, the price of shares is $p$ and the decision in the second period is made in favor of the action and utility $u_i^{-A}(s_i) - ps_i$ if the decision is made not to implement the action. Thus we assume, for simplicity and because any given corporate holding is likely a small fraction of any individual’s lifetime wealth, that utility is quasi-linear in money. We also assume that $u_i^{A'}, u_i^{-A'} > 0 > u_i^{A''}, u_i^{-A''}$ so that individuals value shares but will want to purchase a finite number.

Suppose that, independent of share purchases, the decision is made for $A$ with probability $\pi$. Then if individuals evaluate prospects as maximizers of expected utility, individual $i$’s expected utility is $\pi u_i^A(s_i) + (1 - \pi)u_i^{-A}(s_i) - ps_i$. If individuals can freely choose the number of shares they buy, then their first-order conditions are $\pi u_i^{A'}(s_i) + (1 - \pi)u_i^{-A'}s_i = p$. This must hold for all individuals $i$ and, because we denominate shares as a fraction of the total shares, they must sum to 1.

Suppose that those running the initial IPO, who we will collectively refer to as “the entrepreneur”, wish to choose $\pi$ (or a rule for choosing $\pi$) to maximize profits. The implicit function theorem yields that

$$\left[\pi u_i^{A''}(s_i) + (1 - \pi)u_i^{-A''}(s_i)\right]\frac{ds_i}{d\pi} + u_i^{A'}(s_i) - u_i^{-A}(s_i) = \frac{dp}{d\pi}$$

for all $i$, assuming all individuals buy shares, and $idsid\pi = 0$. Thus, solving out,

$$\frac{dp}{d\pi} = \frac{s_i\epsilon_i}{\bar{\epsilon}} [u_i^{A'}(s_i) - u_i^{-A}(s_i)],$$

where the price elasticity of share purchases,

$$\epsilon_i \equiv \frac{p}{s_i[\pi u_i^{A''}(s_i) + (1 - \pi)u_i^{-A''}(s_i)]} > 0$$

for all $i$ by our assumptions and $\bar{\epsilon}$ is the share-weighted average price elasticity of share purchases across individuals. Thus the initial shareholders want to move toward whichever decision maximizes a weighted average across shareholders of their marginal utility for additional shares, where weights are proportional to the number of shares an individual owns multiplied by their elasticity of their share purchases with respect to price. This is effectively a collective version of the classic Ramsey price discriminatory rule. The problem is that, once shares have been sold, individuals’ interests are determined only by $u_i^A(s_i) - u_i^{-A}(s_i)$, the level of their utility, not by their marginal utility for additional shares. While these may be aligned under some assumptions, in general they will not be perfectly so. For example, individuals might be happy to see a corporation reduce pollution, even at the cost of some profits per share, but this would not make them more willing to pay for the shares of the corporation. In a more subtle but interesting example, a firm taking on a project that is risky but highly profitable might make shareholders better off, but actually less willing to pay for shares at the margin, depending on the nature of risk preferences.
This fundamental problem stops QV, or any ex-post efficient mechanism, from maximizing ex-ante shareholder value. However it is not just ex-post efficient mechanisms that will fail to maximize ex-ante shareholder value: an ex-post inefficient mechanism like voting is likely to do even worse as there is no reason at all that what is preferred by the majority of shares should align with efficiency. For example if shareholders believe, as is possible under majority voting, that one or a few individuals will buy enough shares to vote for the inefficient expropriation of minority shareholders, the equilibrium price of votes will fall to zero as the marginal value of shares is 0 in the state when such an expropriation takes place. More broadly, Jehiel and Moldovanu show that the only information that a mechanism can elicit from agents is that information that directly affects their payoffs from the action that the mechanism will determine.\(^{42}\) That is, any ex-post mechanism followed by free trading in shares (so that the mechanism cannot force agents to rearrange their share holdings) can never elicit complete information about which decision maximizes ex-ante shareholder value. We believe that maximizing ex-post efficiency is likely to line up with ex-ante shareholder value more reliably under QV than under any other mechanism in a broader range of cases and that ex-post efficiency is desirable in itself. However, there will no doubt be cases when ex-post inefficient actions might nonetheless promote ex-ante shareholder value.

Another argument we discuss in the text is that linking votes to shares might improve the share price. To show why we are skeptical of this claim, imagine that \(\pi\) is now a function of the number of shares each individual owns. From the perspective of individual \(i\), only the dependence of \(\pi\) on her own shares is decision-relevant and thus we abuse notation slightly by writing \(\pi si\). Individual \(i\)’s decision-relevant utility is then \(\pi(s_i)u_i^A(s_i) + [1 - \pi(s_i)]u_i^{-A}(s_i) - ps_i\). Now individual \(i\)’s first-order condition is

\[
\pi u_i^{A'}(s_i) + (1 - \pi)u_i^{-A'}(s_i) + \pi'(s_i)[u_i^{A}(s_i) - u_i^{-A}(s_i)] = p.
\]

Thus selling influence over the decision can raise the price of shares to the extent that the most marginal influence is granted to those who gain the most (ex-post) by influencing the decision. This seems to indicate that those who already have a lot of shares (and thus a lot at stake) should be offered the most influence in exchange for purchasing an additional share. However, three things are worth noting.

First, there is no sense in which this component of the price is maximized by the one-share-one-vote rule, or by any simple linking of votes to shares. Thus the current system is hardly geared to raising maximal revenue off of votes.

Second, a series of recent theorems by most notably Mailath and Postlewaite and most generally Al-Najjar and Smorodinsky imply that as \(N\) grows large, so long as each \(si\) becomes small (no one owns a large part of the firm), \(\pi\) must become small for all individuals \(i\), so much so that the amount of revenue that can be raised in total is very small compared to the total value of the firm.\(^{43}\) Thus the revenue raised by linking voting to shares is likely to be very small compared to the value created by getting the decision “right”. While again, the “right” decision from the perspective of ex-ante shareholder value need not be the ex-post efficient decision, this seems (as argued above) likelier than any other ex-post rule to maximize ex-ante shareholder value.


Finally, note that the contribution to the price of the voting rights enters linearly into the equation. Thus, especially given the logic above, it seems sensible to consider what voting system (separate from shares) would raise maximal revenue (even if this is very small, perhaps even irrelevant, by the previous discussion) and thus maximally raise the value of shares (based on distributing this revenue back to shareholders to raise their willingness-to-pay for shares).

Weyl has considered this question in work that is not publicly available but is available on request. He calibrates a model of preferences that depend on both income and an idiosyncratic component, where income follows the log-normal distribution with parameters approximating the income distribution in the United States and an idiosyncratic component to preferences that makes individuals on average willing to pay roughly $\frac{1}{1000}$ of their annual income to make the decision go their way. He considers vote-buying rules where the price of votes are proportional to $\nu x$ where $\nu$ is the number of votes purchased. $x = 2$ is QV, $x = 1$ is linear vote buying (as occurs with shares) and $x = \infty$ is democracy. The figure above shows revenues as a function of $x$. The peak does not occur at 2, but slightly below. However, 2 is very close to the peak and far above either the values of 1 or $\infty$. Thus QV seems likely to raise at very least as much revenue from the allocation of votes as does share-linked voting, likely much more than that and close to the maximal revenue one could achieve by selling off votes to maximize revenue at least in plausible circumstances.

Appendix B: QV’s Bias Against Small-Group Opportunism

The theoretical results Weyl (2013) establishes about QV depend on the assumption that every individual is “small” in the sense that he or she has independently and identically distributed preferences and there are a large number of individuals. In corporate governance, on the other hand, there may be a single individual or a small group of individuals who have strong preferences potentially opposite to those of the large mass of the population. For example, an outside raider or do-gooder, or the CEO, may seek to take an action that is against the interest of most (by numbers of people, not necessarily number of shares) shareholders. Such opportunism, anticipated ex-ante, will lower the willingness of individuals to pay for shares and thus, by the logic of Appendix A, lower the profits. In this appendix we show that such an attempt by a
single individual, which we label “opportunism”, is never advantageous. The individual would always do better to make payments to the other shareholders such that the transaction is in the others’ interests rather than to attempt to vote in the opportunistic scheme themselves. In this sense, QV is opportunism-proof.

We consider only the case when there is a single opportunistic individual. The same argument could easily be extended so long as the size of the opportunistic group is small (of constant size even as the size of other voters/shareholders grows). In particular, suppose that an individual proposes an opportunistic action $A$. There is a large number of shareholders $N$ who have values drawn independently and identically from some distribution with negative mean $\mu$ and variance $\sigma^2$ for action $A$ occurring; they thus typically are harmed by the opportunistic action.\footnote{Note that the iid assumption is not important here as long as all individuals are small. For example, if different individuals had different shares and these scaled up their values, but values were otherwise iid, as long as no individual had a large number of shares the analysis would proceed along precisely the same lines.}

Let $M_i$ be the $i$th moment of the distribution of small individual valuations; we assume the first four moments of the distribution exist. The opportunist has a (for simplicity commonly known) utility $U > 0$ from $A$ being undertaken. For large $N$, $A$ is weakly efficient if and only if $U \geq -N\mu$ and inefficient otherwise. To analyze this case we draw heavily on the techniques of Weyl (2013) without introducing them pedagogically here. Those interested in following the argument more closely should first read at least Sections 1 and 2 of that paper.

Following the logic of Subsection 2.2, we approximate the votes purchased by any small individual with utility $u$ by $v(u) = a(N)u + b(N)u^2$. The mean of votes purchased by all small individuals together is then $m \equiv Na(N)\mu +Nb(N)(\mu^2 + \sigma^2)$ and their variance $s^2 \equiv N(a^2(N)[\mu^2 + \sigma^2] + a(N)b(N)[M_3 - \mu(\mu^2 + \sigma^2)] + b^2(N)[M_4 - (\mu^2 + \sigma^2)^2])$. The distribution of the sum of all $N$ small individuals votes is approximately normal with mean $m$ and variance $s^2$, while the distribution of the sum of any $N-1$ individuals votes is approximately normal with mean $m - \frac{m^2}{N}$ and variance $\frac{s^2}{N}$.

Let $g$ be the distribution of the sum of all $N$ small individual votes. The opportunist will buy votes satisfying $v^*_o = g(-v^*_o)U \approx \frac{e^{(v^*_o + m)^2}}{2s^2} - U$. The distribution of votes by everyone else facing any small individual is approximately normal and has mean $m + v^*_o$ and variance $\frac{s^2}{N}$ because $U$ is common knowledge and thus $v^*_o$ deterministic (or essentially 0 if the opportunist plays a mixed strategy). Thus, by the logic of the proof of Lemma 2 in the Appendix of Weyl (2013),

$$b(N) = \frac{N}{N-1}v^*_o + m \quad \frac{2s^2}{2s\sqrt{2\pi}(N-1)}$$

and

$$a(N) = \frac{p(N)\sqrt{N}e^{-\frac{(N-1)v^*_o + m)^2}{2s^2(N-1)}}}{2s\sqrt{2\pi(N-1)}}$$

where $p(N)$ is the probability that the opportunist tries to buy the election if she is playing a mixed strategy (and 1 if she always tries).
There are three cases to consider: that $\frac{v_0^*}{m}$ heads towards 0 for large $N$, that as $N$ grows large, $v_0^*$ grows in absolute value relative to $m$ without bound and that $\frac{v_0^*}{m}$ converges to a constant.

Let us consider these cases in order. First, if $\frac{v_0^*}{m}$ shrinks as $N$ grows large, clearly the opportunist wins with vanishing probability (given that $s$ shrinks relative to $m$ by the law of large numbers) as $N$ grows large, so our case is already proven.

In the second case, $\begin{align*} a(N) & \to \frac{p(N)e^{-\frac{N(v_0^*)^2}{2(N-1)s^2}}}{2s\sqrt{2\pi}}, \\ b(N) & \to -\frac{v_0^*}{2s^2}, \end{align*}$ and $v_0^* \to \frac{e^{-\frac{(v_0^*)^2}{2s^4}}}{2s\sqrt{2\pi}}U.$

Thus $\begin{align*} m & \to \frac{Np(N)e^{-\frac{N(v_0^*)^2}{2(N-1)s^2}}}{2s\sqrt{2\pi}}\left[\mu - \frac{v_0^*}{2s^2}(\mu^2 + \sigma^2)\right] \\ \text{and} \quad \frac{m}{v_0^*} & \to \frac{Np(N)e^{-\frac{(v_0^*)^2}{2(N-1)s^2}}}{U}\left[\mu - \frac{v_0^*}{2s^2}(\mu^2 + \sigma^2)\right]. \end{align*}$

Clearly in order for our hypothesis that $\frac{m}{v_0^*}$ vanishes for large $N$ to be maintained either $p(N)$ must vanish (in which case clearly there is no limiting chance of the opportunist's victory) or $\frac{(v_0^*)^2}{2(N-1)s^2}$ must explode in $N$. But

\[s^2 \geq Na^2(N)(\mu^2 + \sigma^2) \Rightarrow \frac{p^2(N)Ne^{-\frac{N(v_0^*)^2}{2(N-1)s^2}}}{8s^2\pi} \Rightarrow s = \Omega\left(e^{-\frac{N(v_0^*)^2}{4(N-1)s^2}}\right)\]

Thus $v_0^* = O\left(e^{-\frac{(N-2)(v_0^*)^2}{4(N-1)s^2}}\right)$. But this implies that the number of votes purchased by the opportunist dies exponentially with $N$. This clearly cannot be an equilibrium as any other shareholder would then pay to outvote the opportunist on her own. It can be shown that there is no other equilibrium supported by such behavior for reasons tightly analogous to those in the constant $\frac{v_0^*}{m}$ case to which we now turn.

If $\frac{v_0^*}{m}$ approaches a constant then again there are three subcases, though only one that is interesting. If $\frac{v_0^*}{m} \to \gamma < 1$ then the probability of victory conditional on buying votes by the opportunist is less than $\frac{1}{2}$, which is ruled out by the fact that, as shown in Subsection 2.3 of
Weyl,\textsuperscript{45} such a strategy will never be optimal for the opportunist. If \( \gamma > 1 \) then a logic very similar to the one above in the case where \( \frac{v_0}{m} \) explodes shows this cannot be an equilibrium. In the case where \( \frac{v_0}{m} \to 1 \), we can index events by the finite value to which \( \frac{N}{N-1} \frac{v_0 + m}{s} \) converges to; if the value is infinite then again the analysis resembles the case when \( \frac{v_0^*}{m} \) explodes and is thus omitted. Call the finite limiting value \( \bar{\gamma} \). Then 

\[
\alpha(N) \to \frac{p(N) \sqrt{N} e^{-\frac{\gamma^2(N-1)}{2N}}}{2s\sqrt{2\pi(N-1)}}
\]

and

\[
s^2 \geq Na^2(N)(\mu^2 + \sigma^2) \to \frac{p^2(N) e^{-\frac{\gamma^2(N-1)}{2N}}}{8s^2\pi} \Rightarrow s = \Omega\left(\sqrt[4]{Np(N)}\right).
\]

As above, if \( p(N) \to 0 \) as for large \( N \), then we are done. Suppose this is not the case. Then because \( \frac{b(N)}{a(N)} \to 0 \) as \( N \) grows large in this case by the arguments in the proof of Lemma 2 in Weyl (2013), it is also the case that \( s = O\left(\sqrt{N}\right) \). Thus \( a(N) = \Omega\left(\frac{1}{\sqrt{N}}\right) \). Thus

\[
m \leq Na(N)\mu = \Omega\left(\frac{\mu N^3}{\sqrt{N}}\right).
\]

But \( \frac{v_0^*}{m} \to 1 \) implies that \( v_0^* = \Omega\left(\mu N^3\right) \). Because the cost of votes is quadratic, this means the opportunist would have to be expending an amount on votes that is \( \Omega\left(\mu^2 N^{1/2}\right) \). Clearly this is only worthwhile if \( U = \Omega\left(\mu^2 N^{1/2}\right) \) as the best the opportunist can do is switch the decision’s outcome for sure at the cost of this many votes. But given that the total utility of the small individuals is only of size \( N\mu \), as \( N\mu \) grows large this requires an unboundedly large amount of surplus to be generated by the opportunist’s desired course of action. For example, suppose there are 10,000,000 shareholders and each has a disutility of $50 on average from the action taking place, as seems plausible for a major corporate decision. They thus in aggregate have a disutility on average of $500 million of the action taking place. Then the opportunist would have to have utility from the decision on the order of $80 trillion for an equilibrium where the opportunist wins with non-vanishing probability to prevail. Clearly such an outcome is not a serious possibility for large decisions.

Thus, the only equilibrium is qualitatively similar to the one discussed in Subsection 2.3 of Weyl.\textsuperscript{46} The opportunist, with probability \( p(N) \) that vanishes with \( N \), buys a substantial probability of winning the decision. But now this probability is generated by a mixed strategy of the opportunist rather than the tail probabilities of the distribution of small individuals. Note that this means that whenever the opportunist buys a substantial number of votes, she is indifferent between doing so and buying no votes at all. Thus her expenditures must equal the fraction of \( U \) equal to the probability with which she, conditional on buying these shares, wins the vote. Given that all of this revenue is distributed to individuals other than the opportunist, all individuals other than the opportunist will be weakly better off as a result of this purchase if and only if it is efficient for the opportunist to win.

\textsuperscript{45} Weyl, supra note __.

\textsuperscript{46} Weyl, supra note __.
So clearly the opportunist has nothing to gain by trying to hijack the process in equilibrium when the transaction is large. A much easier strategy is for the opportunist to adjust the offer to promise the other, small shareholders a utility, which is positive on average by paying them a fraction of $U$ contingent on approving the plan. If $U > -N\mu$ this is always feasible while still leaving positive surplus to the opportunist. For example, the opportunist could promise to pay $\frac{U - N\mu}{2}$ to the small shareholders if the transaction is approved, leaving him with surplus utility $\frac{U - N\mu}{2}$. Because QV is efficient for large $N$, this proposal will be approved (even if the opportunist buys no votes at all!) with probability 1 if there are many small shareholders. This will yield the opportunist $\frac{U - N\mu}{2}$, much better than the 0 she would earn trying to hijack the process. Thus, at least with a large number of shareholders, an efficient opportunist maximizing utility will always choose to “payoff” small shareholders, thus making them better off, rather than trying to defeat them under QV.\(^47\)

Intuitively the reason for this, reflected in the logic of the constant limiting case above, is that the quadratic nature of the cost of votes makes it prohibitively expensive to stand alone against the world, even if the rest of the world has less total utility than you do. It is cheaper simply to induce others to agree with you and allow them to, at much lower costs, vote in the proposal that benefits you. Given that they are diffuse and cannot effectively bargain, it should be particularly easy for the opportunist to offer them the minimum amount necessary to ensure his victory so he can keep the maximum amount of surplus. Small shareholders are still better off as a result of this, however. Thus opportunism is not a concern under QV and the appraisal remedy is therefore unnecessary.

\(^47\) Other strategies exist as well. The opportunist could make such a deal with a subset of shareholders. However, this is more expensive for the opportunist than is the other strategy because of the underdog effect (a demonstration is available on request). Another strategy would be to offer side-payments to non-shareholders, or larger side-payments to small than to large shareholders. This could be effective by expanding the pool of those interested in voting in the opportunist’s interests. However, such strategies could easily be ruled out as tantamount to fraudulent collusion/side-payments in the charter, just as other forms of fraud and collusion would have to face legal sanction. Individuals should be restricted from making side payments that are not divided evenly among shareholders per share.
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